SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore – 35

DEPARTMENT OF MATHEMATICS

UNIT – IV INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

Numerical Integration by Simpson's 1/3 Rule

Simpson's 1/3 Rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\cdots+y_{n-1})+2(y_2+y_4+\cdots+y_{n-2})\right]$$

$$= \frac{h}{3}\left[A+4B+2C\right]$$

where A = Sum of the first & last ordinates

B = Sum of the odd ordinates.

C = Sum of the even ordinates.

(ie) an even number of equal subintervals.

Dividing the range into 10 equal parts, find the value of $\int_0^{\pi/2} \sin x\,dx$ by Simpson's 1/3 rule

Soln:

x :	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$
$y = \sin x$:	0	0.1564	0.3090	0.4540	0.5878	0.7071
x :	$6\pi/20$	$7\pi/20$	$8\pi/20$	$9\pi/20$	$10\pi/20$	
$y : \sin x$:	0.8090	0.8910	0.9511	0.9877	1	

By Simpson's 1/3 rule,

$$\int_0^{\pi/2} \sin x \, dx = \frac{h}{3}\left[(y_0 + y_{11}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8 + y_{10})\right]$$

$$= \frac{\pi}{20} \cdot \frac{1}{3}\left[(0+1) + 4(3.1962) + 2(2.6569)\right]$$

$$= 1.0000$$

Find the value of $\log_e 5$ from $\int_0^5 \frac{dx}{4x+5}$ by Simpson's $1/3$ rule $(n=10)$.

Soln: Here $y(x) = \frac{1}{4x+5}$

$$h = \frac{5-0}{10} = \frac{1}{2} = 0.5$$

x:	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
y:	0.2	0.1429	0.1111	0.0909	0.0769	0.0667	0.0588	0.0526	0.047	0.0434	0.04

By Simpson's $1/3$ rule,

$$\int_0^5 \frac{dx}{4x+5} = \frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+y_6+\cdots) + 4(y_1+y_3+y_5+\cdots)\right]$$

$= \frac{1}{6}[2.4148]$

$= 0.4025$ —— (1)

$\int_0^5 \frac{dx}{4x+5} = \left.\frac{\log(4x+5)}{4}\right|_0^5$

$= \frac{1}{4}(\log 25 - \log 5)$

$= \frac{1}{4}\log\left(\frac{25}{5}\right)$

$= \frac{1}{4}\log 5$ —— (2)

From (1) & (2)

$\Rightarrow \frac{1}{4}\log 5 = 0.4025$ loge

$\Rightarrow \log 5 = 1.61$

Using Simpson's 1/3 rule, Evaluate $\int_0^{1.2} e^{-x^2}dx$, taking $h = 0.2$.

NUMERICAL INTEGRATION BY TRAPEZOIDAL

TRAPEZOIDAL RULE :

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

$$= \frac{h}{2}[A+2B]$$

where A = Sum of the first & last ordinates

B = Sum of the remaining ordinates.

(1) Using trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

Soln: Gn: $y(x) = \frac{1}{1+x^2}$

Here $h = \frac{b-a}{n}$ where $a=-1$, $b=1$, and $n=8$

$\Rightarrow h = \frac{2}{8} = 0.25$

x:	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
y:	0.5	0.64	0.8	0.9412	1	0.9412	0.8	0.64	0.5

Trapezoidal rule,

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\dots+y_{n-1})\right]$$

$= \frac{h}{2}$ [sum of the first and last ordinate + 2 × sum of the remaining ordinates]

$$= \frac{0.25}{2}\left[(0.5+0.5)+2(0.64+0.8+0.9412+0.9412+0.8+0.64)\right]$$

$$= \frac{0.25}{2} \times 12.5248$$

$$= 1.5656$$

(2) Dividing the range into 10 equal parts, find the value of $\int_0^{\pi/2} \sin x\,dx$ by (i) Trapezoidal rule

Soln:

x :	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$
$y=\sin x$:	0	0.1564	0.3090	0.4540	0.5878	0.7071
x :	$6\pi/20$	$7\pi/20$	$8\pi/20$	$9\pi/20$	$10\pi/20$	
$y:\sin x$:	0.8090	0.8910	0.9511	0.9877	1	

By Trapezoidal rule;

$$\int_0^{\pi/2} \sin x\,dx = \frac{h}{2}\left[(y_0+y_{11}) + 2(y_1+y_2+\cdots+y_{10})\right]$$

Here $h = \dfrac{\frac{\pi}{2}-0}{10} = \dfrac{\pi}{20}$

$$= \frac{h}{2}\left[(0+1) + 2(0.1564+0.3090+0.4540+0.5878+0.7071+0.8090+0.8910+0.9511+0.9877)\right]$$

$$= \frac{\pi}{20}\cdot\frac{1}{2}\left[12.7062\right]$$

$$= 0.9980$$