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DEPARTMENT OF MATHEMATICS UNIT – IV INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

NEWTON'S FORWARD AND BACKWARD DIFFERENCE FORMULA

(EQUAL ENTERVALS)

Let the function y = f(n) takes the values y_0, y_1, \dots, y_n at the points n_0, n_1, \dots, n_n where $x_t = n_0 + ih$. Then Newton's Jouward interpolation polynomial is $y(n) = P_n(n) = f(n)$ $= y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \frac{u(u-1)(u-2)\cdots(u-(n-1))}{3!} \Delta^n y_0$ where $u = \frac{n-n_0}{-h}$; this the difference between two intervals.



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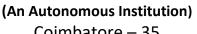
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Then Newton's Backward Öderpolation polynomial
is given by

$$y(x) = P_n(x) = \frac{1}{2}(x)$$

 $= \frac{y_n + \frac{u}{1!} \nabla y_n + \frac{u(u+1)}{2!} \nabla^2 y_{n+1} \frac{u(u+1)(u+2)}{3!} \nabla^3 y_n$
 $+ \dots + u(u+1)(u+2) \dots (u+(n-1)) \nabla^n y_n$
where $u = \frac{y_1 - y_n}{T_n}$
Note: Jorward Backeward.
Jirst order: Jorward Backeward.
 $A'y_0 = \frac{y_1 - y_0}{T_n}$
Second order: $\nabla y_n = \frac{y_{n-1}}{y_{n-1}}$
 $A'y_1 = \frac{y_2 - y_1}{2}$
Second order: $\nabla^2 y_n = \nabla y_n - \nabla y_{n-1}$
 $A'y_0 = Ay_r Ay_0$
 $\nabla^3 y_n = \nabla^2 y_n - \nabla y_{n-1}$
Third order: $\nabla^3 y_n = \nabla^2 y_n - \nabla^2 y_{n-1}$

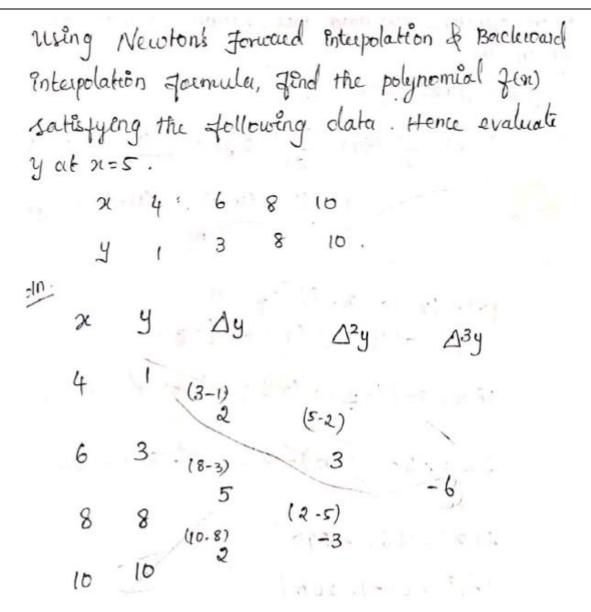






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Jorward Enterpolation : Here 210= 4; 70=1; A=2 $u = \frac{2 - 4}{2}$ $y(x) = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-i)\Delta^2 y_0}{2!} + \frac{u(u-i)(u-a)}{3!} \Delta^3 y_0$ = 1 + $\left(\frac{\chi - 4}{2}\right)(2) + \left(\frac{\chi - 4}{2}\right)\left(\frac{\chi - 4}{2} - 1\right)\frac{(3)}{21} + \frac{(\chi - 4)}{2}$ $\left(\frac{2(-4)}{2}\right)\left(\frac{2(-4)}{2}-1\right)\left(\frac{2(-4)}{2}-2\right)\left(\frac{-6}{2}\right)$ $1 + 2 - 4 + (2 - 4)(2 - 6) \times \frac{3}{2} + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 6)(2 - 8) + (2 - 4)(2 - 8) + (2 - 4)(2 - 8) + (2 - 4)(2 - 8) + (2 - 4)(2 - 8) + (2 - 4)(2 - 8) + (2 - 4)(2 - 8) + (2 - 4)(2 - 8) + (2 - 4)(2 - 8) + (2 - 4)(2 - 8) + (2 - 8)(2 - 8)(2 - 8) + (2 - 8)(2 - 8)(2 - 8) + (2 - 8)(2 - 8)(2 - 8) + (2 - 8)(2$ x-3'+(x-10x+24) = + x3-82+104 x-192x-1 $=\frac{1}{8}\left(8n-24+3n^{2}-30n+72+(-n^{3}+18n^{2}-104n+192)\right)$ $=\frac{1}{8}\left(-x^{3}+21n^{2}-126n+240\right)$ $y(s) = \frac{1}{2} \left(-(s)^3 + 21(s)^2 - 126(s) + 240 \right) = 1.25$ Backward Enterpolation: Here 2n=10; yn=10; h=2. $u = \frac{\alpha - 10}{2}$



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 $\begin{aligned} y(x) &= y_n + \frac{u}{1!} \nabla y_n + \frac{u(u+1)}{2!} \nabla^2 y_n + u(u+1)(u+2) \nabla^3 y_n \\ &= 10 + \left(\frac{n-10}{2}\right)(2) + \left(\frac{n-10}{2}\right) \left(\frac{n-10}{2}+1\right) \left(-\frac{3}{2!}\right)^{\frac{1}{2!}} \\ &\left(\frac{n+10}{2}\right) \left(\frac{n-10}{2}+1\right) \left(\frac{n-10}{2}+2\right) \frac{(-6)}{3!} \\ &= \frac{1}{8} \left(-x^3 + 21n^2 - 126n + 240\right) \\ y(s) &= \frac{1}{8} \left(-(s)^3 + 21(s)^2 - 126(s) + 240\right) \\ &= 1 + 2s^{-1} \end{aligned}$