MODIFIED EULER METHOD :

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{1}{2}h, \; y_0 + \frac{h}{2}f(x_0, y_0)\right)\right] \text{ for } (x_0, y_0)$$

$$y_2 = y_1 + h\left[f\left(x_1 + \frac{h}{2}, \; y_1 + \frac{h}{2}f(x_1, y_1)\right)\right] \text{ for } (x_1, y_1)$$

$$y_{n+1} = y_n + h\left[f\left(x_n + \frac{h}{2}, \; y_n + \frac{h}{2}f(x_n, y_n)\right)\right] \text{ for } (x_n, y_n)$$

for $n = 0, 1, 2, \ldots$

This formula is called modified Euler's formula.

Compute y at $x = 0.25$ by modified Euler method given

$$y' = 2xy, \quad y(0) = 1$$

Sln: $f(x, y) = 2xy$

$$x_0 = 0; \quad y_0 = 1; \quad h = 0.25$$

$$y_1 = y_0 + h f \left[x_0 + \frac{h}{2}, \ y_0 + \frac{h}{2} f(x_0, y_0) \right]$$

$$= 1 + (0.25) f \left[0 + \frac{0.25}{2}, \ 1 + \frac{0.25}{2} \left[2 x_0 y_0 \right] \right]$$

$$= 1 + (0.25) f (0.125, 1.125)$$

$$= 1 + 0.25 \left[2 (0.125)(1) \right]$$

$$= 1 + 0.0625$$

$$= 1.0625$$

① Solve $y' = 1 - y$, $y(0) = 0$ by modified Euler's method with $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$.

Soln:

$$y(0.1) = 0.095$$

$$y(0.2) = 0.1809$$

$$y(0.3) = 0.2587$$