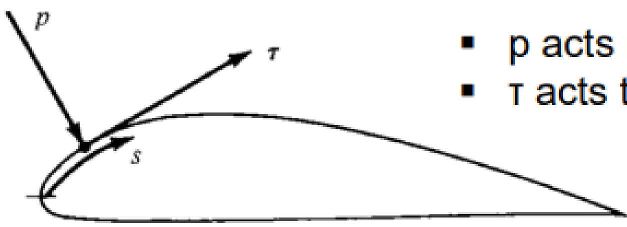
Aerodynamic forces and moments

- At first glance, the generation of the aerodynamic force may seem complex…
- However, in all cases, the aerodynamic forces and moments on the body are due to only two basic sources:

  1. Pressure distribution,
  2. Shear stress distribution over the body surface

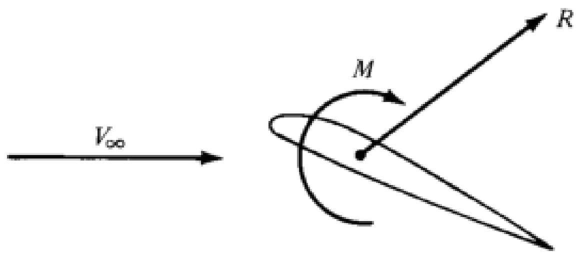


$p = p(s)$ = surface pressure distribution
$\tau = \tau(s)$ = surface shear stress distribution

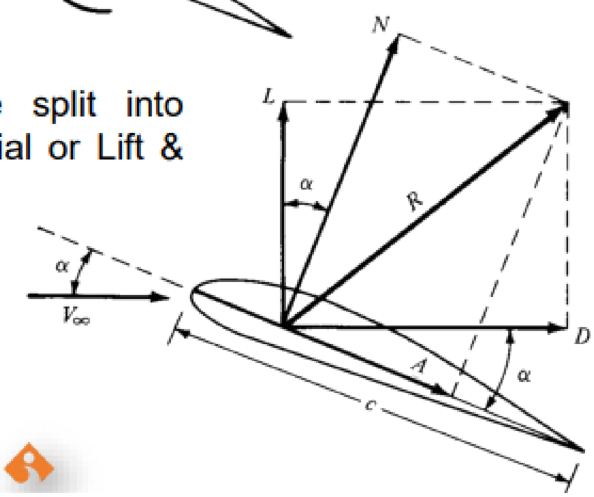
- p acts normal to the surface,
- τ acts tangential to the surface.

Aerodynamic forces and moments

- The net effect of the *p* and *τ* distributions integrated over the complete body surface is a resultant aerodynamic force *R* and moment *M* on the body.

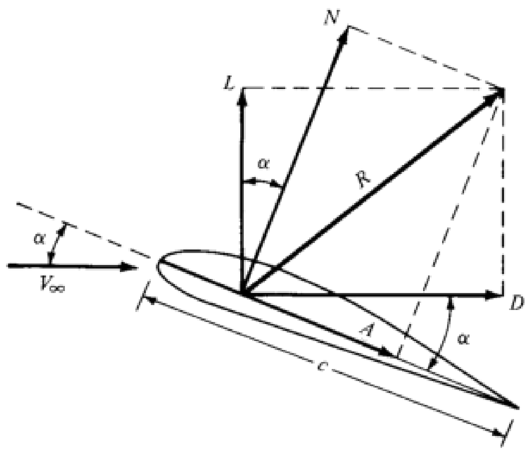


- The resultant *R* can be split into components, Normal & Axial or Lift & Drag forces.



Aerodynamic forces and moments

- The flow far away from the body is called the *freestream,* and hence V_∞ is also called the freestream velocity.
- The chord c is the linear distance from the leading edge to the trailing edge of the body.
- The angle of attack α is defined as the angle between c and V_∞.
- The geometrical relation between these two sets of components is,

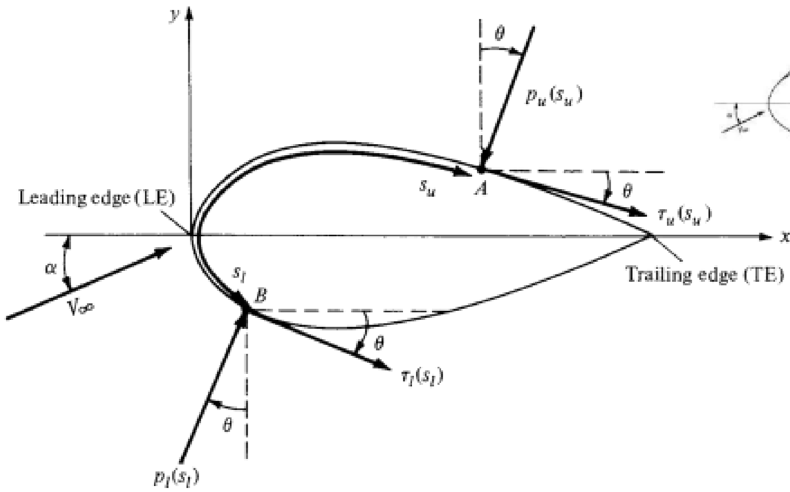


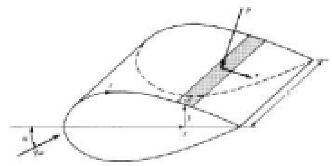
L; component of *R* perpendicular to V_∞
D; component of *R* parallel to V_∞

$$L = N\cos\alpha - A\sin\alpha$$
$$D = N\sin\alpha + A\cos\alpha$$

Aerodynamic forces and moments

- We can examine in more detail the integration of the pressure and shear stress distributions to obtain the aerodynamic forces and moments.





$dS = (ds)(1)$

For the upper body surface,

$$dN'_u = -p_u ds_u \cos\theta - \tau_u ds_u \sin\theta$$

$$dA'_u = -p_u ds_u \sin\theta + \tau_u ds_u \cos\theta$$

For the lower body surface,

$$dN'_l = p_l ds_l \cos\theta - \tau_l ds_l \sin\theta$$

$$dA'_l = p_l ds_l \sin\theta + \tau_l ds_l \cos\theta$$

Aerodynamic forces and moments

- The total normal and axial forces *per unit span* are obtained by integrating equations from the leading edge (LE) to the trailing edge (TE):

$$N' = -\int_{LE}^{TE} (p_u \cos\theta + \tau_u \sin\theta)\, ds_u + \int_{LE}^{TE} (p_l \cos\theta - \tau_l \sin\theta)\, ds_l$$

$$A' = \int_{LE}^{TE} (-p_u \sin\theta + \tau_u \cos\theta)\, ds_u + \int_{LE}^{TE} (p_l \sin\theta + \tau_l \cos\theta)\, ds_l$$

- We can get lift and drag forces based on the previous equations;

$$L = N \cos\alpha - A \sin\alpha$$
$$D = N \sin\alpha + A \cos\alpha$$

Aerodynamic forces and moments

- The aerodynamic moment exerted on the body depends on the point about which moments are taken.
- Lets consider moments taken about the leading edge.
- The moment per unit span about the leading edge due to *p* and *τ* on the elemental area *dS* on the upper and lower surface are

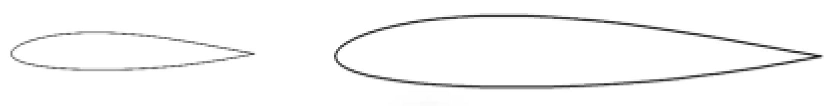
$$dM'_u = (p_u \cos\theta + \tau_u \sin\theta)x\, ds_u + (-p_u \sin\theta + \tau_u \cos\theta)y\, ds_u$$
$$dM'_l = (-p_l \cos\theta + \tau_l \sin\theta)x\, ds_l + (p_l \sin\theta + \tau_l \cos\theta)y\, ds_l$$

- By integrating from the leading to the trailing edges, we obtain the pitching moment about the leading edge per unit span

$$M'_{\text{LE}} = \int_{\text{LE}}^{\text{TE}} [(p_u \cos\theta + \tau_u \sin\theta)x - (p_u \sin\theta - \tau_u \cos\theta)y]\, ds_u$$
$$+ \int_{\text{LE}}^{\text{TE}} [(-p_l \cos\theta + \tau_l \sin\theta)x + (p_l \sin\theta + \tau_l \cos\theta)y]\, ds_l$$

Aerodynamic forces and moments

- In Equations; θ, x, and y are known functions of s for a given body shape.
- A major goal of theoretical or experimental aerodynamics is to calculate $p(s)$ and $\tau(s)$ for a given body shape and freestream conditions.
- We get the aerodynamic forces and moments based on them.
- In aerodynamics, shape is probably the most important factor.
- We may eliminate the scale of the shape by defining some dimensionless coefficients.



Aerodynamic forces and moments

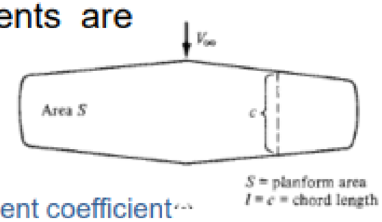
- Let ρ_∞ and V_∞ be the density and velocity, respectively, in the freestream, far ahead of the body.

- We define a dimensional quantity called the freestream *dynamic pressure* as

$$q_\infty \equiv \frac{1}{2}\rho_\infty V_\infty^2$$

- In addition, let *S* be a reference area and *l* be a reference length.

- The dimensionless force and moment coefficients are defined as follows:



$$C_L \equiv \frac{L}{q_\infty S}$$ Lift coefficient

$$C_D \equiv \frac{D}{q_\infty S}$$ Drag coefficient

$$C_M \equiv \frac{M}{q_\infty S l}$$ Moment coefficient

Aerodynamic forces and moments

- For two-dimensional bodies, it is conventional to denote the aerodynamic coefficients by lowercase letters; for example,

$$c_l \equiv \frac{L'}{q_\infty c} \quad c_d \equiv \frac{D'}{q_\infty c} \quad c_m \equiv \frac{M'}{q_\infty c^2}$$

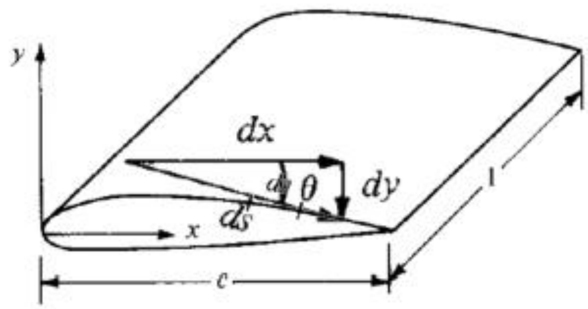
- Two additional dimensionless quantities of immediate use are

Pressure coefficient $C_p \equiv \frac{p - p_\infty}{q_\infty}$

Skin friction coefficient $c_f \equiv \frac{\tau}{q_\infty}$

p local static pressure
p_∞ static free stream pressure
q_∞ dynamic free stream pressure

- From the geometry



$$dx = ds \cos\theta$$
$$dy = -(ds \sin\theta)$$
$$S = c(1)$$

Aerodynamic forces and moments

$$C_N \equiv \frac{N}{q_\infty S}$$

$$C_A \equiv \frac{A}{q_\infty S}$$

- We obtain the following integral forms for the force and moment coefficients

$$c_n = \frac{1}{c}\left[\int_0^c (C_{p,l} - C_{p,u})\, dx + \int_0^c \left(c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right) dx\right]$$

$$c_a = \frac{1}{c}\left[\int_0^c \left(C_{p,u}\frac{dy_u}{dx} - C_{p,l}\frac{dy_l}{dx}\right) dx + \int_0^c (c_{f,u} + c_{f,l})\, dx\right]$$

$$c_{m_{LE}} = \frac{1}{c^2}\left[\int_0^c (C_{p,u} - C_{p,l}) x\, dx - \int_0^c \left(c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right) x\, dx \right.$$
$$\left. + \int_0^c \left(C_{p,u}\frac{dy_u}{dx} + c_{f,u}\right) y_u\, dx + \int_0^c \left(-C_{p,l}\frac{dy_l}{dx} + c_{f,l}\right) y_l\, dx\right]$$

- The lift and drag coefficients can also be obtained:

$$c_l = c_n \cos\alpha - c_a \sin\alpha$$
$$c_d = c_n \sin\alpha + c_a \cos\alpha$$