Two mark Questions and Answers

1.State Lagrange's interpolation formula.

Solution:

Let y=f(x) be a function which takes the values $y_0, y_1, y_2, \ldots\ldots, y_n$ corresponding to x= $x_0, x_1, x_2, \ldots., x_n$.

Then,Lagrange's interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots\ldots.(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots\ldots.(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\ldots\ldots.(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots\ldots.(x_1-x_n)}y_1 + \ldots\ldots\ldots$$
$$+\frac{(x-x_0)(x-x_1)\ldots\ldots.(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots\ldots.(x_n-x_{n-1})}y_n$$

2.What is Inverse interpolation?

Solution:

Inverse interpolation is the process of finding the value of x corresponding to a value of y,not present in the table.

3.Give the inverse of lagrange's interpolation formula.

Solution:

$$x = \frac{(y-y_1)(y-y_2)\ldots\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots\ldots(y_0-y_n)}x_0 + \frac{(y-y_0)(y-y_2)\ldots\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots\ldots(y_1-y_n)}x_1 + \ldots\ldots\ldots.$$
$$+\frac{(y-y_0)(y-y_1)\ldots\ldots(y-y_{n-1})}{(y_n-y_1)(y_n-y_1)\ldots\ldots(y_n-y_{n-1})}x_n$$

4.What is the advantage of Lagrange's formula?

Solution:Lagrange's interpolation formula can be used whether the values of x,the independent variable are equally spaced or not whether the difference of y become smaller or not.

5. Derive Newton's backward difference formula by using operator method.[Gregory Newton's backward difference interpolation formula].

Solution:

$$y(x) = f(x) = P_n(x)$$
$$= y_n + \frac{u}{1!}\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \ldots\ldots..$$
$$+\frac{u(u+1)(u+2)\ldots..(u+(n-1))}{n!}\nabla^n y_n$$

where $u = \frac{x-x_n}{h}$

6. State Gregory Newton's forward difference interpolation formula.

Solution:

$$y(x)=f(x)=P_n(x)$$
$$=y_0+\frac{u}{1!}\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0+\dots\dots\dots$$
$$+\frac{u(u-1)(u-2).....(u-(n-1))}{n!}\Delta^n y_0$$
$$\text{where}\quad u=\frac{x-x_0}{h}$$

7. Given f(0)=-2,f(1)=2 and f(2)=8.Find the polynomial using Newton's interpolation formula.

Solution:

x	y=f(x)	$\Delta f(x)$	$\Delta^2 f(x)$
0	-2	4	
1	2	6	2
2	8		

By Newton's forward difference interpolation formula

$$y(x)=y_0+\frac{u}{1!}\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0+\dots\dots\dots$$
$$+\frac{u(u-1)(u-2).....(u-(n-1))}{n!}\Delta^n y_0$$
$$\text{where}\quad u=\frac{x-x_0}{h}$$

Here

$$x_0=0, h=1\Rightarrow u=x$$
$$\therefore y(x)=-2+\frac{x}{1!}(4)+\frac{x(x-1)}{2!}(2)$$
$$=-2+4x+x(x-1)$$
$$=x^2+3x-2.$$

8.State the merits and demerits of Newton's forward and backward interpolation formula.

Merits:

Newton's forward and backward interpolation formula are applicable for interpolation near the beginning and end respectively of tabulated values.

Demerits:

Newton's forward and backward interpolation formula used only for equal intervals (or) equidistant intervals.

9.Using Newton's forward difference formula,write the formula for the first,second and third order derivatives at $x=x_0$

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_0}=\frac{1}{h}\left[\Delta y_0-\frac{1}{2}\Delta^2 y_0+\frac{1}{3}\Delta^3 y_0-\frac{1}{4}\Delta^4 y_0+\dots\dots\right]$$
$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0}=\frac{1}{h^2}\left[\Delta^2 y_0-\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0-\frac{5}{6}\Delta^5 y_0+\dots\dots\right]$$
$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0}=\frac{1}{h^3}\left[\Delta^3 y_0-\frac{3}{2}\Delta^4 y_0+\dots\dots\right]$$

10. Construct an Newton difference table for the points (0,-1),(1,1),(2,1) and (3,-2).

Solution:

Difference table :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	-1			
		1+1:2		
1	1		0-2:-2	
		1-1:0		-3+2:-1
2	1		-3-0:-3	
		-2-1:-3		
3	-2			

11. State Trapezoidal rule.

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\right] \text{ where } h = \frac{b-a}{n}$$

$$= \frac{h}{2}[A + 2B]$$

where A=sum of the first and last ordinates & B=sum of the remaining ordinates.

12. Using Newton's backward difference formula,write the formula for the first,second and third order derivatives at $x = x_n$

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \dots\dots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \dots\dots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + \dots\dots\right]$$

13. Find $\frac{dy}{dx}$ at x=1 from the following table:

x	1	2	3	4
y	1	8	27	64

Solution:

y	Δ	Δ^2	Δ^3

1 8 27 64	7 19 37	12 18	6

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0\right]$$

$$= (1)\left[7 - \frac{12}{2} + \frac{6}{3}\right] = 3$$

14.State Simpson's 3/8 rule.

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_{5+} .... + y_{n-1}) + 2(y_3 + y_6 + ... + y_{n-3})\right] \text{ where } h = \frac{b-a}{n}$$

15.State Simpson's 1/3 rule.

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})\right] \text{ where } h = \frac{b-a}{n}$$

$$= \frac{h}{3}[A + 4B + 2C]$$

where A=sum of the first and last ordinates.
B=sum of the odd ordinates
C= sum of the even ordinates

16.Evaluate $\int_{1/2}^{1} \frac{1}{x}dx$ by Trapezoidal rule,dividing the range into 4 equal parts.

Solution:

$$h = \frac{1 - \frac{1}{2}}{4} = \frac{1}{8}$$

x	1/2	5/8	6/8	7/8	8/8
f(x)	8/4	8/5	8/6	8/7	8/8

$$\therefore \int_{1/2}^{1} \frac{1}{x}dx = \frac{1}{8}.\frac{1}{2}\left[\left(\frac{8}{4} + \frac{8}{8}\right) + 2\left(\frac{8}{5} + \frac{8}{6} + \frac{8}{7}\right)\right]$$

$$= \frac{1}{16}\left[3 + 2\left(\frac{856}{210}\right)\right] = \frac{1171}{1680} = 0.6971$$

17.Evaluate $\int_{0}^{1} \frac{dx}{1+x^2}$ with h=1/6 by Trapezoidal rule.

Solution:

Given $f(x)=\int_0^1 \frac{dx}{1+x^2}, h=1/6$

x	0	1/6	2/6	3/6	4/6	5/6	6/6
f(x)	1	36/37	9/10	4/5	9/13	36/31	1/2

$$\therefore \int_0^1 \frac{dx}{1+x^2} = \frac{1}{12}\left[\left(1+\frac{1}{2}\right)+2\left(\frac{36}{37}+\frac{9}{10}+\frac{4}{5}+\frac{9}{13}+\frac{36}{31}\right)\right]$$
$$= 0.7842$$

18.Using Simpson,s rule(one-third) evaluate $\int_0^1 xe^x dx$ taking 4 intervals.Compare your result with actual value.

Solution:

Given $f(x)=xe^x; h=\frac{1}{4}=0.25$

x	0	0.25	0.5	0.75	1
f(x)	1	0.321	0.824	1.588	2.718

By Simpson's 1/3 rule

$$\therefore \int_0^1 xe^x dx = \frac{0.25}{3}\left[(0+2.718)+2(0.321+1.588)+4(0.524)\right]$$
$$= 0.819 = 1$$

By actual integration

$$\int_0^1 xe^x dx = [xe^x - e^x]_0^1 = e^1 - e^1 - (0-1) = 1$$

19. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Simpson,s 3/8 rule &check by actual integration.

Solution:

Given $f(x)=\frac{1}{1+x^2}; h=\frac{6-0}{6}=1$

x	0	1	2	3	4	5	6
f(x)	1	0.500	0.200	0.100	0.0588	0.0384	0.2702

By Simpson's 3/8 rule,

$$\therefore \int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+.....+y_{n-1})+2(y_3+y_6+...+y_{n-3})\right]$$
$$= \frac{3}{8}\left[(1+0.2702)+3(0.5+0.2+0.0588+0.0384)+2(0.1)\right]$$
$$= 1.3570$$

By actual integration,

$$\int_0^6 \frac{dx}{1+x^2} = \left[\tan^{-1}(x)\right]_0^6 = \tan^{-1}(6) = 1.4056$$

20. What are the errors in Trapezoidal & Simpson's rules of numerical integration?

Solution:

Error in Trapezoida rule $|E| < \frac{(b-a)}{12} h^2 M; order \quad is \quad h^2$

Error in Simpson's rule $|E| < \frac{(b-a)}{180} h^4 M; order \quad is \quad h^4$