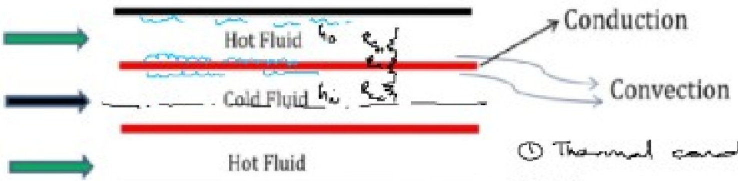
SNS COLLEGE OF TECHNOLOGY, COIMBATORE-35

DEPARTMENT OF MECHANICAL ENGINEERING, 16ME306/ Heat and Mass Transfer – **UNIT III - PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS**

Topic - Types of Heat Exchangers- Heat Exchanger Analysis- LMTD Method

Heat exchangers are the devices that facilitate heat transfer between two (or) more fluids. at different temperatures, while keeping them from mixing (or) without mixing. Heat transfer in heat exchanger involves convection in each fluid and conduction through the wall separating the fluids.

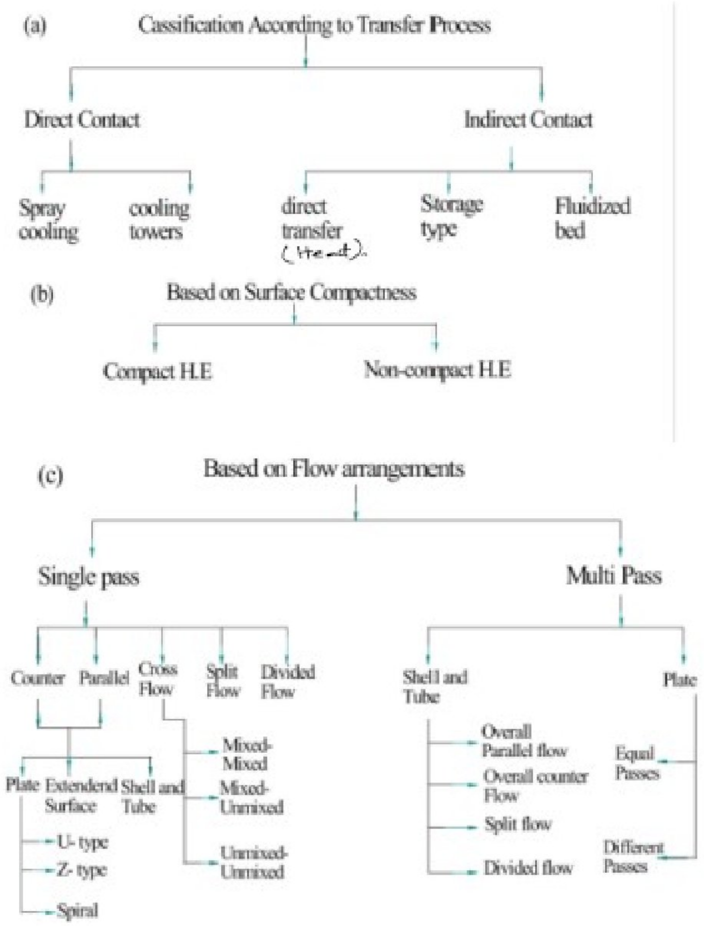


$Q = \dot{m} C_p \Delta T$

① Thermal conductivity
② C_p.
③ ΔT

Examples: Shell and tube heat exchanger.
Boilers.
Cooling towers
Car radiators etc.,

Classification of Heat Exchangers:



Heat transfer analysis of heat exchangers

To design (or) predict the performance, the fluid temperatures, overall heat transfer coefficient and surface area parameters are to be evaluated. They are modelled as steady-flow devices. The methods used for analysis are:

1) LMTD approach (Log mean Temp difference)

known → Temperatures, Mass flow rates, overall heat transfer coefficient.

Calculate the size of the heat exchanger D, L, Q

2) ε-NTU approach! (Number of Transfer units).

known → Either inlet (or) outlet temperatures, size.

Type and size of heat exchanger.

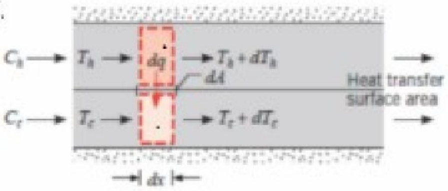
Calculate the missing temperatures, Load on the heat exchanger.

LMTD for parallel flow heat exchanger

Heat capacity:

$C_h = \dot{m}_h C_{ph}$.

$C_c = \dot{m}_c C_{pc}$.



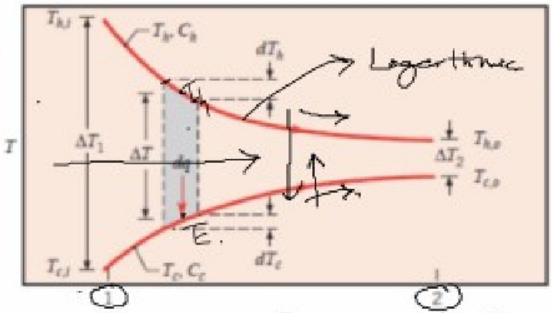
i = inlet //

o = outlet //

Temp Vs dist.

$\Delta T_1 = T_{hi} - T_{ci}$

$\Delta T_2 = T_{ho} - T_{co}$.



$T_{hi} > T_{ho}$.

$T_{co} > T_{ci}$

Assumptions: [Both parallel & counter flow HEs] ⊗

1) Steady flow.

2) KE and PE are neglected.

3) Thermo physical properties are constant over the entire length of HE.

4) No heat loss to the surrounding (adiabatic)

5) Heat transfer coefficient is constant over the entire length.

Let, A = Surface area in m^2.

$\dot{m}_c, \dot{m}_h$ = Mass flow rate in kg/s.

$\Delta T = T_h - T_c$ = Local temperature difference b/n hot and cold fluids in °C (or) K.

U = Overall heat transfer coefficient.

Applying the energy balance: (hot & cold fluid)

$$Q = \underbrace{\dot{m}_c C_{pc}}_{C_c} (T_{co} - T_{ci}) = \underbrace{\dot{m}_h C_{ph}}_{C_h} (T_{hi} - T_{ho}) \rightarrow ①$$

Heat capacities are.

$$C_c = \dot{m}_c C_{pc} \text{ and } C_h = \dot{m}_h C_{ph}. \rightarrow ②$$

Rate of heat transfer 'dQ' from the hot to the cold fluid through an elemental area 'dA' is given by;

$$dQ = U.dA\,dT \rightarrow ③ \qquad [Q = UA\Delta T].$$

Also, dQ should be equal to the heat given up by the hot fluid (or) gained by the cold fluid flowing through an elemental area 'dA', is.

$$dQ = \dot{m}_c C_{pc}\, dT_c = -\dot{m}_h C_{ph}\, dT_h \rightarrow ④$$

[−ve sign due to heat lost by the hot fluid] ⊗

Rewriting the above equations

$$dT_c = \frac{dQ}{\dot m_c C_{pc}} \quad \text{and} \quad dT_h = -\frac{dQ}{\dot m_h C_{ph}} \rightarrow \textcircled{5}$$

If, $\Delta T_1 = T_{hi} - T_{ci}$ and $\Delta T_2 = T_{ho} - T_{co}$, then.

$$dT_h - dT_c = -\frac{dQ}{\dot m_h C_{ph}} - \frac{dQ}{\dot m_c C_{pc}}.$$

$$d(T_h - T_c) = -dQ\left[\frac{1}{\dot m_h C_{ph}} + \frac{1}{\dot m_c C_{pc}}\right] \rightarrow \textcircled{6}.$$

Substitute ④ in ⑥, we get.

$$d(T_h - T_c) = -U\,dA\,dT\left[\frac{1}{\dot m_h C_{ph}} + \frac{1}{\dot m_c C_{pc}}\right].$$

Integrating between inlet and outlet.

$$\int_{\text{inlet}=\Delta T_1}^{\text{outlet}=\Delta T_2} \frac{d(T_h - T_c)}{dT} = -\int_{\text{inlet}}^{\text{out}} U.dA\left[\frac{1}{\dot m_h C_{ph}} + \frac{1}{\dot m_c C_{pc}}\right]$$

$$\ln\left(\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}}\right) = -UA\left[\frac{1}{\dot m_h C_{ph}} + \frac{1}{\dot m_c C_{pc}}\right].$$

Substituting for $\dot m_h C_{ph}$ & $\dot m_c C_{pc}$ from ①, we get.

$$\ln\left(\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}}\right) = -UA\left[\frac{T_{hi} - T_{ho}}{Q} + \frac{T_{co} - T_{ci}}{Q}\right]$$

$$\ln\left(\frac{T_{ho}-T_{co}}{T_{hi}-T_{ci}}\right) = \frac{-UA}{Q}\left[T_{hi}-T_{ho}+T_{co}-T_{ci}\right]$$

Rearranging the term.

$$\ln\left(\frac{T_{ho}-T_{co}}{T_{hi}-T_{ci}}\right) = \frac{-UA}{Q}\left[(T_{hi}-T_{ci})-(T_{ho}-T_{co})\right]$$

$$\ln\left(\frac{T_{ho}-T_{co}}{T_{hi}-T_{ci}}\right) = \frac{UA}{Q}\left[(T_{ho}-T_{co})-(T_{hi}-T_{ci})\right].$$

$$\therefore Q = \frac{UA\left[(T_{ho}-T_{co})-(T_{hi}-T_{ci})\right]}{\ln\left(\frac{T_{ho}-T_{co}}{T_{hi}-T_{ci}}\right)}$$

$$Q = \frac{UA(\Delta T_2-\Delta T_1)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

[OR]

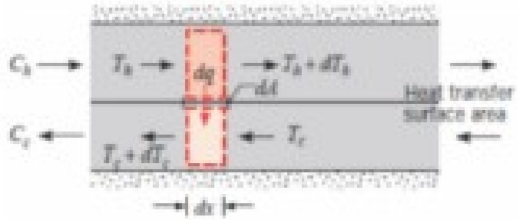
$$Q = \frac{UA(\Delta T_1-\Delta T_2)}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

→ Logarithmic mean temperature diff (LMTD)

LMTD for Counter flow heat exchanger.

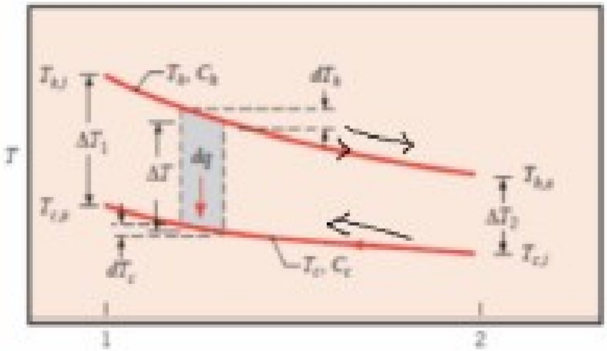
$L \rightarrow l$

$l \rightarrow L$



$\Delta T_1 = T_{hi} - T_{co}$

$\Delta T_2 = T_{ho} - T_{ci}$



Let, A = Surface area in m^2.

$\dot{m}_c, \dot{m}_h$ = Mass flow rate in kg/s.

$\Delta T = T_h - T_c$ = Local temperature difference b/n hot and cold fluids in °C (or) K.

U = Overall heat transfer coefficient.

Applying the energy balance: (hot & cold fluid)

$$Q = \underbrace{\dot{m}_c C_{pc}}_{C_c} (T_{co} - T_{ci}) = \underbrace{\dot{m}_h C_{ph}}_{C_h} (T_{hi} - T_{ho}) \rightarrow ①$$

Heat capacities are.

$$C_c = \dot{m}_c C_{pc} \text{ and } C_h = \dot{m}_h C_{ph}. \longrightarrow ②$$

Rate of heat transfer 'dQ' from the hot to the cold fluid through an elemental area 'dA' is given by;

$$dQ = U.dA\,dT \rightarrow ③ \qquad [Q = UA\Delta T].$$

Also, dQ should be equal to the heat given up by the hot fluid (or) gained by the cold fluid flowing through an elemental area 'dA', is.

$$\otimes \quad dQ = -\dot{m}_c C_{pc}\,dT_c = -\dot{m}_h C_{ph}\,dT_h \rightarrow ④$$

[−ve sign due to heat lost for hot fluid.
−ve sign due to opposite direction for cold fluid]

Rewriting the above equations

$$dT_c = \frac{-dQ}{\dot{m}_c C_{pc}} \quad \text{and} \quad dT_h = \frac{-dQ}{\dot{m}_h C_{ph}} \rightarrow ⑤$$

If, $\Delta T_1 = T_{hi} - T_{co}$ and $\Delta T_2 = T_{ho} - T_{ci}$, then.

$$dT_h - dT_c = -\frac{dQ}{\dot{m}_h C_{ph}} + \frac{dQ}{\dot{m}_c C_{pc}}.$$

$$d(T_h - T_c) = -dQ\left[\frac{1}{\dot{m}_h C_{ph}} - \frac{1}{\dot{m}_c C_{pc}}\right] \rightarrow ⑥ .$$

Substitute Ⓘ in Ⓔ, we get.

$$d(T_h - T_c) = -U\,dA\,dT\left[\frac{1}{\dot m_h C_{ph}} - \frac{1}{\dot m_c C_{pc}}\right].$$

Integrating between inlet and outlet.

$$\int_{\text{inlet}=\Delta T_1}^{\text{outlet}=\Delta T_2} \frac{d(T_h - T_c)}{dT} = -\int_{\text{inlet}}^{\text{out}} U.\,dA\left[\frac{1}{\dot m_h C_{ph}} - \frac{1}{\dot m_c C_{pc}}\right]$$

$$\ln\left(\frac{T_{ho} - T_{ci}}{T_{hi} - T_{co}}\right) = -UA\left[\frac{1}{\dot m_h C_{ph}} - \frac{1}{\dot m_c C_{pc}}\right].$$

Substituting for $\dot m_h C_{ph}$ & $\dot m_c C_{pc}$ from ①, we get

$$\ln\left(\frac{T_{ho} - T_{ci}}{T_{hi} - T_{co}}\right) = -UA\left[\frac{T_{hi} - T_{ho}}{Q} - \frac{T_{co} - T_{ci}}{Q}\right]$$

$$\ln\left(\frac{T_{ho} - T_{ci}}{T_{hi} - T_{co}}\right) = -\frac{UA}{Q}\left[T_{hi} - T_{ho} - T_{co} + T_{ci}\right]$$

Rearranging the term.

$$\ln\left(\frac{T_{ho} - T_{ci}}{T_{hi} - T_{co}}\right) = -\frac{UA}{Q}\left[(T_{hi} - T_{co}) - (T_{ho} - T_{ci})\right].$$

$$\ln\left(\frac{T_{ho}-T_{ci}}{T_{hi}-T_{co}}\right) = \frac{UA}{Q}\left[(T_{ho}-T_{ci})-(T_{hi}-T_{co})\right].$$

$$\therefore Q = \frac{UA\left[(T_{ho}-T_{ci})-(T_{hi}-T_{co})\right]}{\ln\left(\frac{T_{ho}-T_{co}}{T_{hi}-T_{ci}}\right)}$$

$$Q = \frac{UA\,(\Delta T_2-\Delta T_1)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

[OR] $$Q = \frac{UA\,(\Delta T_1-\Delta T_2)}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

→ Logarithmic mean temperature diff (LMTD)

ⓧ Note:

Even though the mathematic expression for both parallel and counter flow heat exchanger looks similar, the temperature differences (ΔT) at the inlet and outlet are different.

References:

1. Kothandaraman C.P “Fundamentals of Heat and Mass Transfer” New Age International, New Delhi,4th Edition 2012 (Unit I, II, III, IV, V).
2. Frank P. Incropera and David P. DeWitt, “Fundamentals of Heat and Mass Transfer”, John Wiley and Sons, New Jersey,6th Edition1998(Unit I,II,III,IV, V)
3. MIT open courseware – https://ocw.mit.edu/courses/mechanical-engineering

Other web sources