Topic - Tutorial- film wise condensation

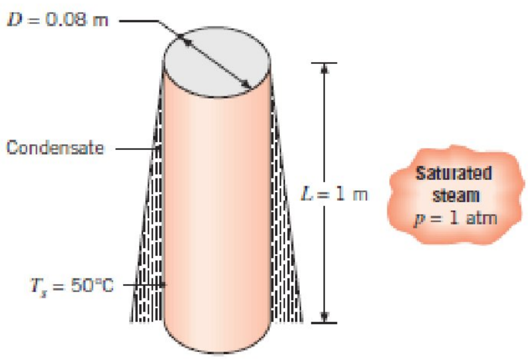
The outer surface of a vertical tube, which is 1 m long and has an outer diameter of 80 mm, is exposed to saturated steam at atmospheric pressure and is maintained at 50°C by the flow of cool water through the tube. What is the rate of heat transfer to the coolant, and what is the rate at which steam is condensed at the surface?

SOLUTION

Known: Dimensions and temperature of a vertical tube experiencing condensation of steam at its outer surface.

Find: Heat transfer and condensation rates.

Schematic:



Assumptions:

1. The condensate film thickness is small relative to the cylinder diameter.
2. Negligible concentration of noncondensable gases in the steam.

Properties: Table A.6, saturated vapor (p = 1.0133 bars): $T_{sat} = 100°C$, $\rho_v = 1/v_g = 0.596\ kg/m^3$, $h_{fg} = 2257\ kJ/kg$. Table A.6, saturated liquid ($T_f = 75°C$): $\rho_l = 1/v_f = 975\ kg/m^3$, $\mu_l = 375 \times 10^{-6}\ N \cdot s/m^2$, $k_l = 0.668\ W/m \cdot K$, $c_{p,l} = 4193\ J/kg \cdot K$, $\nu_l = \mu_l/\rho_l = 385 \times 10^{-9}\ m^2/s$.

Analysis: Since we assume the film thickness is small relative to the cylinder diameter, we may use the correlations of Sections 10.7 and 10.8. With

$$Ja = \frac{c_{p,l}(T_{sat} - T_s)}{h_{fg}} = \frac{4193\ J/kg \cdot K(100 - 50)\ K}{2257 \times 10^3\ J/kg} = 0.0929$$

it follows that

$$h'_{fg} = h_{fg}(1 + 0.68\,Ja) = 2257\ kJ/kg\ (1.0632) = 2400\ kJ/kg$$

From Equation 10.42,

$$P = \frac{k_l L (T_{sat} - T_s)}{\mu_l h'_{fg} (\nu_l^2/g)^{1/3}}$$

$$= \frac{0.668\ W/m \cdot K \times 1\ m \times (100 - 50)\ K}{375 \times 10^{-6}\ N \cdot s/m^2 \times 2.4 \times 10^6\ J/kg \left[\dfrac{(385 \times 10^{-9}\ m^2/s)^2}{9.8\ m/s^2}\right]^{1/3}} = 1501$$

Therefore, Equation 10.44 applies:

$$\overline{Nu}_L = \frac{1}{P}(0.68\,P + 0.89)^{0.82} = \frac{1}{1501}(0.68 \times 1501 + 0.89)^{0.82} = 0.20$$

Then

$$\overline{h}_L = \frac{\overline{Nu}_L k_l}{(\nu_l^2/g)^{1/3}} = \frac{0.20 \times 0.668\ W/m \cdot K}{\left[\dfrac{(385 \times 10^{-9}\ m^2/s)^2}{9.8\ m/s^2}\right]^{1/3}} = 5300\ W/m^2 \cdot K$$

and from Equations 10.33 and 10.34

$$q = \overline{h}_L(\pi D L)(T_{sat} - T_s) = 5300\ W/m^2 \cdot K \times \pi \times 0.08\ m \times 1\ m\ (100 - 50)\ K = 66.6\ kW \quad \triangleleft$$

$$\dot{m} = \frac{q}{h'_{fg}} = \frac{66.6 \times 10^3\ W}{2.4 \times 10^6\ J/kg} = 0.0276\ kg/s \quad \triangleleft$$

Note that using Equation 10.26, with the corrected latent heat, the film thickness at the bottom of the tube $\delta(L)$ for the wave-free laminar assumption is

$$\delta(L) = \left[\frac{4k_l\mu_l(T_{sat} - T_s)L}{g\rho_l(\rho_l - \rho_v)h'_{fg}}\right]^{1/4}$$

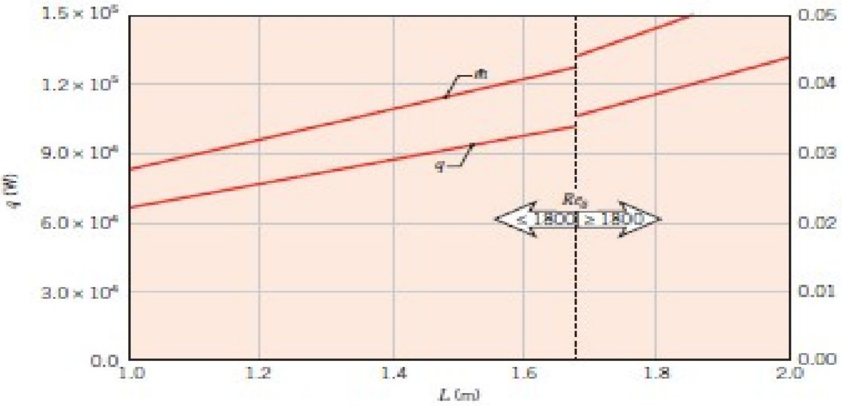
$$\delta(L) = \left[\frac{4 \times 0.668\ \text{W/m}\cdot\text{K} \times 375 \times 10^{-6}\ \text{kg/s}\cdot\text{m}\ (100 - 50)\ \text{K} \times 1\ \text{m}}{9.8\ \text{m/s}^2 \times 975\ \text{kg/m}^3 (975 - 0.596)\ \text{kg/m}^3 \times 2.4 \times 10^6\ \text{J/kg}}\right]^{1/4}$$

$$\delta(L) = 2.18 \times 10^{-4}\ \text{m} = 0.218\ \text{mm}$$

Hence $\delta(L) \ll (D/2)$, and use of the vertical plate correlation for a vertical cylinder is justified.

Comments:

1. The condensation heat and mass rates may be increased by increasing the length of the tube. For $1 \le L \le 2$ m, the calculations yield the variations shown below, for which $1000 \le Re_\delta \le 2330$ or $1500 \le P \le 3010$. The foregoing calculations were performed by using the wavy-laminar correlation, Equation 10.44, for $P \le 2530$ ($L \le 1.68$ m) and Equation 10.45, for $P > 2530$ ($L > 1.68$ m). Note, however, that the correlations do not provide equivalent results at $P = 2530$. In particular, Equation 10.45 is a function of Pr, whereas Equation 10.44 is not.



The tube bank of a steam condenser consists of a square array of 400 tubes, each of diameter $D = 2r_1 = 6$ mm.

1. If horizontal, unfinned tubes are exposed to saturated steam at a pressure of $p = 0.15$ bar and the tube surface is maintained at $T_s = 25°C$, what is the rate at which steam is condensed per unit length of the tube bank?
2. If annular fins of height $h = r_2 - r_1 = 1$ mm, thickness $t = 1$ mm, and pitch $S = 2$ mm are added, determine the minimum condensation rate per unit length of tubing.

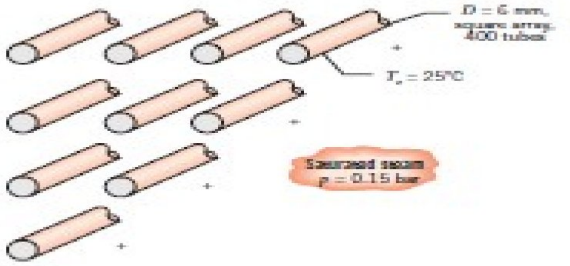
SOLUTION

Known: Configuration and surface temperature of unfinned and finned condenser tubes exposed to saturated steam at 0.15 bar.

Find:

1. Condensation rate per unit length of unfinned tubing.
2. Minimum condensation rate per unit length of finned tubing.

Schematic:



Assumptions:

1. Spatially uniform cylinder and fin temperature.
2. Average heat transfer coefficient varies with tube row with $n = -1/6$ in Equation 10.49.
3. Negligible concentration of noncondensable gases in the steam.

Properties: Table A.6, saturated vapor ($p = 0.15$ bar): $T_{sat} = 327$ K $= 54$°C, $\rho_v = 1/v_g = 0.098$ kg/m^3, $h_{fg} = 2373$ kJ/kg, $\sigma = 0.0671$ N/m. Table A.6, saturated water ($T_f = 312.5$ K): $\rho_l = 1/v_f = 992$ kg/m^3, $\mu_l = 663 \times 10^{-6}$ N·s/m^2, $k_l = 0.631$ W/m·K, $c_{p,l} = 4178$ J/kg·K.

Analysis:

1. Equation 10.46 may be rearranged to yield an expression for the convection coefficient for the top, unfinned tube which is of the form

$$\overline{h}_D = C\left[\frac{\rho_l g(\rho_l - \rho_v)k_l^3 h'_{fg}}{\mu_l(T_{sat} - T_s)D}\right]^{1/4}$$

where $C = 0.729$ for a tube and

$$h'_{fg} = h_{fg}(1 + 0.68\,Ja) = h_{fg} + 0.68c_{p,l}(T_{sat} - T_s)$$
$$= 2373 \times 10^3 \text{ J/kg} + 0.68 \times 4178 \text{ J/kg}\cdot\text{K} \times (327 - 298)\text{ K}$$
$$= 2455 \text{ kJ/kg}$$

Therefore

$$\overline{h}_D = 0.729\left[\frac{992 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times (992 - 0.098)\text{ kg/m}^3 \times (0.631 \text{ W/m}\cdot\text{K})^3 \times 2455 \times 10^3 \text{ J/kg}}{633 \times 10^{-6} \text{ kg/s}\cdot\text{m} \times (327 - 298)\text{ K} \times 6 \times 10^{-3}\text{ m}}\right]^{1/4}$$
$$= 11{,}120 \text{ W/m}^2\cdot\text{K}$$

From Equation 10.49 the array-averaged convection coefficient is

$$\bar{h}_{D,N} = \bar{h}_D N^n = 11{,}120\ \text{W/m}^2\cdot\text{K} \times 20^{-1/6} = 6747\ \text{W/m}^2\cdot\text{K}$$

From Equation 10.34 the condensation rate per unit length of tubing is

$$\dot{m}'_{uft} = N \times N \frac{\bar{h}_{D,N}(\pi D)(T_{sat} - T_s)}{h'_{fg}}$$

$$= 20 \times 20 \times 6747\ \text{W/m}^2\cdot\text{K} \times \pi \times 6 \times 10^{-3}\ \text{m} \times (327 - 298)\ \text{K}/2455 \times 10^3\ \text{J/kg}$$

$$= 0.601\ \text{kg/s}\cdot\text{m} \quad \triangleleft$$

2. From Equation 10.48, the minimum enhancement attributable to the annular fins is

$$\varepsilon_{fi,min} = \frac{q_{fi,min}}{q_{uft}} = \frac{\dot{m}'_{fi,min}}{\dot{m}'_{uft}} = \frac{tr_2}{Sr_1}\left[\frac{r_1}{r_2} + 1.02\frac{\sigma r_1}{(\rho_l - \rho_v)gt^3}\right]^{1/4}$$

$$= \frac{1 \times 4}{2 \times 3}\left[\frac{3}{4} + 1.02\frac{0.0671\ \text{N/m} \times 3 \times 10^{-3}\ \text{m}}{(992 - 0.098)\ \text{kg/m}^3 \times 9.8\ \text{m/s}^2 \times (1 \times 10^{-3}\ \text{m})^3}\right]^{1/4}$$

$$= 1.44$$

Therefore, the minimum condensation rate for the finned tubes is

$$\dot{m}'_{fi,min} = \varepsilon_{fi,min}\dot{m}'_{uft} = 1.44 \times 0.601\ \text{kg/s}\cdot\text{m} = 0.866\ \text{kg/s}\cdot\text{m} \quad \triangleleft$$

Comment: A value of $n = -1/6$ was used in Equation 10.49. However, for finned tubes the value of n is expected to be between zero and $-1/6$. For $n = 0$, the condensation rate per unit length of tubing would be

$$\dot{m}'_{fi,min} = \varepsilon_{fi,min} \times N \times N \frac{\bar{h}_D(\pi D)(T_{sat} - T_s)}{h'_{fg}}$$

$$= 1.44 \times 20 \times 20 \times 11{,}120\ \text{W/m}^2\cdot\text{K} \times \pi \times 6 \times 10^{-3}\ \text{m} \times (327 - 298)\ \text{K}/2455 \times 10^3\ \text{J/kg}$$

$$= 1.43\ \text{kg/s}\cdot\text{m}$$

The preceding rate is for a *nonoptimized* condition where condensate fills the entire inter-fin region. Actual enhancements of $\varepsilon_{fi,max} \approx 4$ might be expected [50]. For $\varepsilon_{fi,max} = 4$ and $n = 0$, the condensation rate would be

$$\dot{m}'_{fi} = \varepsilon_{fi,max} \times N \times N \frac{\bar{h}_D(\pi D)(T_{sat} - T_s)}{h'_{fg}}$$

$$= 4 \times 20 \times 20 \times 11{,}120\ \text{W/m}^2\cdot\text{K} \times \pi \times 6 \times 10^{-3}\ \text{m} \times (327 - 298)\ \text{K}/2455 \times 10^3\ \text{J/kg}$$

$$= 3.96\ \text{kg/s}\cdot\text{m}$$

Hence the condensation rate could potentially be increased by $100 \times (3.96 - 0.601)$ kg/s·m/ 0.601 kg/s·m = 560% by using finned tubes.

References:

1. Kothandaraman C.P "Fundamentals of Heat and Mass Transfer" New Age International, New Delhi,4th Edition 2012 (Unit I, II, III, IV, V).
2. Frank P. Incropera and David P. DeWitt, "Fundamentals of Heat and Mass Transfer", John Wiley and Sons, New Jersey,6th Edition1998(Unit I,II,III,IV, V)
3. MIT open courseware – https://ocw.mit.edu/courses/mechanical-engineering

Other web sources