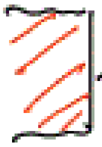
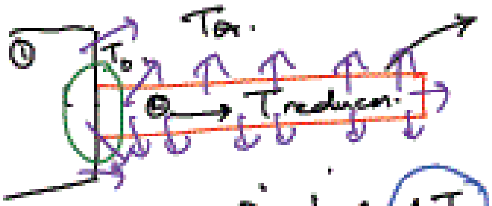
Extended surfaces (or) Fins.



T_∞. Augment the rate of heat transfer from the object

Solution:



Additional area is added to increase the surface area.

$$\dot{Q} = h A (\Delta T.)$$

$A \uparrow \rightarrow Q \uparrow$

Say example: $T_0 = 100°C$ $T_\infty = 30°C$

T reduces along the length of fin

$A \Delta T \uparrow \rightarrow$ Use of fin is effective.

$A \Delta T \downarrow \rightarrow$ Use of fin is not beneficial.

1-D Analysis of fins. Why?

$$q_{across} \sim k_{fin} \frac{(T_c - T_s)}{\delta}$$

$$\sim h (T_s - T_\infty)$$

So. $$\frac{k_{fin} (T_c - T_s)}{\delta} \sim h (T_s - T_\infty)$$

$$\frac{(T_c - T_s)}{(T_s - T_\infty)} \sim \frac{h.\delta}{k_{fin}}$$

1-D analysis is possible if.

$$(T_c - T_s) << (T_s - T_\infty)$$

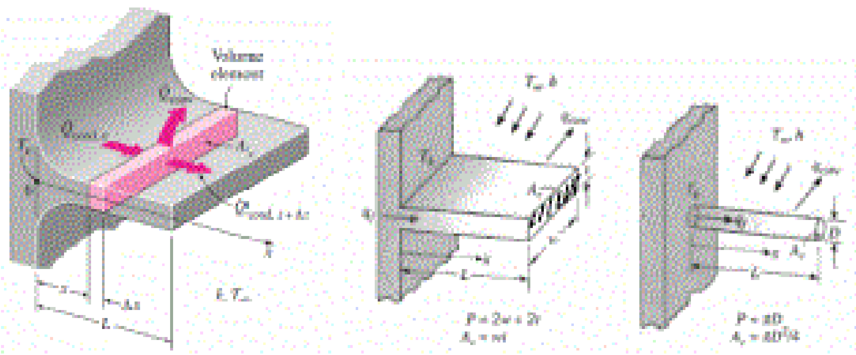
$$\boxed{\frac{h\delta}{k_{fin}}} << 1.$$

Biot No (Bi)

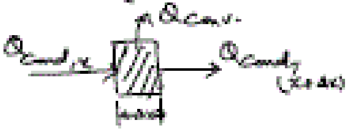
Biot Number $Bi = \dfrac{\delta / k_{fin} A}{1/hA} = \dfrac{\text{Conductive resistance}}{\text{Convective resistance.}}$

<u>Derivation of 1-D fin-equation with uniform C/s</u>

$T(x) = ?$ $T(y)$ is neglected.



(a) (b) Rectangular C/s (c) Circular C/s



Base is @ temperature 'T_0', or 'T_b'

For the volume, Energy balance.

$$E_{in} + \cancel{E_{gen}}^{0} - E_{out} = \cancel{E_{stored}}^{0} \longrightarrow (1)$$

For steady state with no heat generation

$$E_{in} = E_{out} \quad (E_{gen} \ \& \ E_{stored} = 0)$$

Rate of heat conduction @ x = Rate of heat cond @ (x+Δx) + Rate of heat convection from element

$$Q_{cond}\Big|_x = Q_{cond}\Big|_{x+\Delta x} + Q_{conv} \longrightarrow (2)$$

$$Q_{cond}\Big|_x = -kA_c \frac{dT}{dx} \quad \left(\text{Fourier equation}\right)$$

$$Q_{cond}\Big|_{x+\Delta x} = -kA_c\frac{dT}{dx} - kA_c\frac{d^2T}{dx^2}\Delta x$$

$$Q_{conv} = h.\underbrace{P\Delta x}_{A_s}(T - T_\infty)$$

A_c = c/s area.
T = Local Temp
T_∞ = Amb Temp
$A_{surf} = P\Delta x$

(2) ⇒

$$-kA_c\frac{dT}{dx} = -kA_c\frac{dT}{dx} - kA_c\frac{d^2T}{dx^2}\Delta x + hP\Delta x\, dT$$

(fin material)
k = Thermal Conductivity
P = Perimeter

$$kA_c\frac{d^2T}{dx^2} - hP\,(T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0 \qquad m^2 = \frac{hP}{kA_c}$$

$$\frac{d^2T}{dx^2} - m^2\underbrace{(T - T_\infty)}_{\theta} = 0 \longrightarrow (3)$$

Let, $\theta = T - T_\infty$

$\frac{d\theta}{dT} = 1$. $\therefore d\theta = dT$

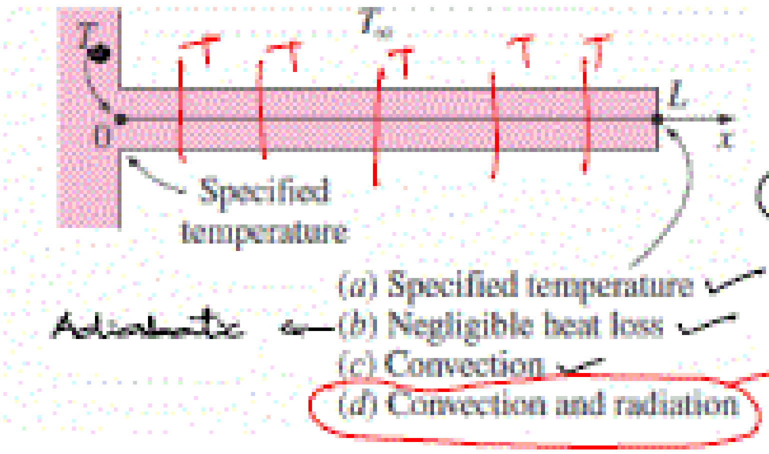
$\therefore$ ① becomes;

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \rightarrow ②$$ (Linear, homogeneous second order)

General solution is of the for;

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \longrightarrow ③$$

C_1 & C_2 are the constants obtained from B.C's.



L = 10cm

L = 0.1, T_1

L = 0.4, T_2

($\theta = T - T_\infty$)

→ Special case

Case 1: Long fin ($T_{x=L} = T_\infty$) (x=∞) $\theta = T - T_\infty$.

At the end fin temperature is equal to the ambient temperature.

@ $x = 0$ $\theta = T_b - T_\infty = \theta_b$. $T = T_b$,

@ $x = \infty$ (L) $\theta = T_\infty - T_\infty = 0$ $T = T_\infty$.

[Equilibrium]

① $\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$ $\quad [\because e^0 = 1]$

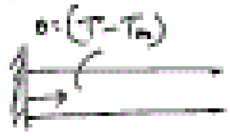
$\theta_b = C_1 e^0 + C_2 e^{-0}$

$\theta_b = C_1 + C_2 \longrightarrow ③$

② $0 = C_1 e^{m\infty} + C_2 e^{-m\infty}$ $\quad [e^{-\infty} = 0]$

$0 = C_1 \longrightarrow ④$

$\theta_b = C_2$ $\quad \therefore C_1 = 0$



Substituting C_1 & C_2 in eqn ②

$\theta(x) = 0 e^{mx} + \theta_b \cdot e^{-mx}$

$$\left.\begin{aligned} \therefore \frac{\theta(x)}{\theta_b} &= e^{-mx} \\ \text{(or)} \quad \frac{T(x) - T_\infty}{T_b - T_\infty} &= e^{-mx} \end{aligned}\right\} \longrightarrow ⑤$$

Temp distribution for along fin

$T(x) = (T_b - T_\infty) e^{-mx} + T_\infty$.

Heat flow through the fin is given by;

$$Q = \int_{L=0}^{L=\infty} h\,P(T - T_\infty)\,dx = -kA \left.\frac{dT}{dx}\right|_{x=0}$$

Substituting for $(T - T_\infty)$ from ⑤, we get.

$$Q = \int_{L=0}^{L=\infty} h\,P\,(T_b - T_\infty)\,e^{-mx}\,dx.$$

$$Q = hP(T_b - T_\infty) \cdot \frac{-1}{m} \cdot \left[e^{-mx}\right]_0^{\infty}$$

$$Q = hP(T_b - T_\infty) - \frac{1}{m}[0-1] \qquad m = \sqrt{\frac{hP}{KA}}$$

$$Q = \sqrt{hPKA_c}\,(T_b - T_\infty) \longrightarrow (6)$$

Case 2:- Fin of finite length & end insulated (short fin).

[Fin of negligible tip area is assumed to be adiabatic]

B.C $\quad \theta(x) = C_1 e^{mx} + C_2 e^{-mx}$

@ $x = 0$, $T = T_b$; $T_b - T_\infty = \theta_b$

@ $x = L$ $\left.\frac{d\theta}{dx}\right|_{x=L} = 0$ $\left[\because \frac{dT}{dx} = 0\right]$ } (1)

From (1) condition $\theta_b = C_1 + C_2 \rightarrow$ (same).

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

$$\frac{d\theta}{dx} = m.C_1 e^{mx} - m.C_2 e^{-mx} \qquad mL = \text{Constant}$$

From (2) condition, we get. $\qquad m = \sqrt{\frac{hP}{KA}}$

$$0 = mC_1 e^{mL} - mC_2 e^{-mL} \qquad m^2 = \frac{hP}{KA}.$$

$$\cancel{m}.C_1 e^{mL} = \cancel{m}C_2 e^{-mL}$$

$$C_1 = \frac{C_2 e^{-mL}}{e^{mL}}$$

$$C_1 = C_2 e^{-2mL} \longrightarrow (2)$$

But, $C_1 + C_2 = \theta_b$

$$C_2 e^{-2mL} + C_2 = \theta_b.$$

$$C_2[1 + e^{-2mL}] = \theta_b.$$

$$C_2 = \frac{\theta_b}{(1 + e^{-2mL})} \longrightarrow (3)$$

\

Heat transfer through the fin

$$Q = -kA \frac{dT}{dx}\bigg|_{x=0} = \int_0^L hP(T-T_\infty)dx.$$

$$\frac{dT}{dx} = \left[-\frac{m \sinh m(L-x)}{\cosh mL}\right]\theta_b.$$

$$\frac{dT}{dx}\bigg|_{x=0} = \theta_b\left[\frac{-m \sinh mL}{\cosh mL}\right] = \left[-m \cdot \tanh mL\right]\theta_b$$

$$\therefore Q = -kA\left[-m \cdot \tanh mL\right]\theta_b.$$

$$Q = kA\sqrt{\frac{hP}{kA}} \tanh mL \cdot \theta_b.$$

Challenge

$Q \propto \tanh mL$

$$\therefore Q = \sqrt{hPkA} \cdot \tanh mL \cdot \theta_b \longrightarrow (2)$$

Case 3: Fin with end not insulated (Convection)

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

B.C (1) @ $x=0$, $T=T_b$; $\theta_b = T_b - T_\infty$; (Same).

B.C (2) @ $x=L$ $\quad -kA \frac{d\theta}{dx}\Big|_{x=L} = h.A(T-T_\infty)\Big|_{x=L}$ $\therefore \theta = T - T_\infty$

From the first BC; $\theta_b = C_1 + C_2 \Rightarrow C_2 = \theta_b - C_1 \rightarrow (1)$

$$\theta = C_1 e^{mx} + C_2 e^{-mx}.$$

$$\frac{d\theta}{dx} = m.C_1 e^{mx} - m C_2 e^{-mx}$$

$$\frac{d\theta}{dx}\bigg|_{x=L} = m C_1 e^{mL} - m. C_2 e^{-mL}.$$

$$-k\cancel{A}\left[m C_1 e^{mL} - m C_2 e^{-mL}\right]_{x=L} = h\cancel{A}\left[C_1 e^{mL} + C_2 e^{-mL}\right]_{x=L}$$

$$\left[-C_1 e^{mL} + C_2 e^{-mL}\right] = \frac{h}{Km}\left[C_1 e^{mL} + C_2 e^{-mL}\right]$$

Substituting for C_2 from equation ① $C_2 = \theta_b - C_1$

$$-C_1 e^{mL} + (\theta_b - C_1) e^{-mL} = \frac{h}{mK}\left[C_1 e^{mL} + (\theta_b - C_1) e^{-mL}\right].$$

$$-C_1 e^{mL} + \theta_b e^{-mL} - C_1 e^{-mL} = \frac{h}{mK}\left[C_1 e^{mL} + \theta_b e^{-mL} - C_1 e^{-mL}\right]$$

$$-C_1 (e^{mL} + e^{-mL}) + \theta_b e^{-mL} = \frac{h}{mK}\left[C_1 (e^{mL} - e^{-mL}) + \theta_b e^{-mL}\right]$$

$$\theta_b e^{-mL}\left[1 - \frac{h}{mK}\right] = C_1\left[(e^{mL} + e^{-mL}) + \frac{h}{mK}(e^{mL} - e^{-mL})\right]$$

③
+x −
x = L

$$C_1 = \frac{\theta_b e^{-mL}(1 - h/mK)}{(e^{mL} + e^{-mL}) + \frac{h}{mK}(e^{mL} - e^{-mL})} \longrightarrow ②$$

$$\therefore C_2 = \theta_b - C_1$$

$$= \theta_b - \frac{\theta_b e^{-mL}(1 - h/mK)}{(e^{mL} + e^{-mL}) + \frac{h}{mK}(e^{mL} - e^{-mL})}$$

$$C_2 = \frac{\theta_b\left[(e^{mL} + e^{-mL}) + \frac{h}{mK}(e^{mL} - e^{-mL})\right] - \theta_b e^{-mL}(1 - h/mK)}{(e^{mL} + e^{-mL}) + \frac{h}{mK}(e^{mL} - e^{-mL})}$$

$$C_2 = \frac{\theta_b e^{mL} + \overset{①}{\cancel{\theta_b e^{-mL}}} + \theta_b \frac{h}{mK} e^{mL} - \overset{②}{\cancel{\theta_b \frac{h}{mK} e^{-mL}}} - \overset{①}{\cancel{\theta_b e^{-mL}}} + \overset{②}{\cancel{\theta_b \frac{h}{mK} e^{-mL}}}}{(e^{mL} + e^{-mL}) + \frac{h}{mK}(e^{mL} - e^{-mL})}$$

$$C_2 = \frac{\theta_L e^{mL}\left(1 + \frac{h}{mK}\right)}{\left(e^{mL} + e^{-mL}\right) + \frac{h}{mK}\left(e^{mL} - e^{-mL}\right)} \longrightarrow (2)$$

Substituting for C_1 & C_2 in the solution eqn.

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta(x) = \frac{\left[\theta_L e^{-mL}\left(1 - \frac{h}{mK}\right)\right]e^{mx} + \left[\theta_L e^{mL}\left(1 + \frac{h}{mK}\right)\right]e^{-mx}}{\left(e^{mL} + e^{-mL}\right) + \frac{h}{mK}\left(e^{mL} - e^{-mL}\right)}$$

$$\frac{\theta(x)}{\theta_L} = \frac{e^{-m(L-x)} - \frac{h}{mK} e^{-m(L-x)} + e^{m(L-x)} + \frac{h}{mK} e^{m(L-x)}}{\left(e^{mL} + e^{-mL}\right) + \frac{h}{mK}\left(e^{mL} - e^{-mL}\right)}$$

$$\frac{\theta(x)}{\theta_L} = \frac{\left[e^{m(L-x)} + e^{-m(L-x)}\right] + \frac{h}{mK}\left[e^{m(L-x)} - e^{-m(L-x)}\right]}{\left(e^{mL} + e^{-mL}\right) + \frac{h}{mK}\left(e^{mL} - e^{-mL}\right)}$$

(*) $\left[\cosh x = \frac{e^{x} + e^{-x}}{2} \quad \& \quad \sinh x = \frac{e^{x} - e^{-x}}{2}\right]$

(*) $\left\{ \frac{\theta(x)}{\theta_L} = \frac{T(x) - T_\infty}{T_L - T_\infty} = \frac{\cosh m(L-x) + \frac{h}{mK}\sinh m(L-x)}{\cosh mL + \frac{h}{mK}\sinh mL} \right.$

Heat transfer is given by.

$$Q = -KA\left.\frac{d\theta}{dx}\right|_{x=0} = hA\left(T - T_\infty\right)\Big|_{x=0}$$

$$\theta = \left[\frac{\cosh m(L-x) + \frac{h}{mK}\sinh m(L-x)}{\cosh mL + \frac{h}{mK}\sinh mL}\right]\theta_L$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = \frac{\left[-m \sinh mL - \frac{h}{mK} \cosh mL\right]\theta_b}{\cosh mL + \frac{h}{mK} \sinh mL} \qquad \div \cosh mL$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = \frac{\left[-m \tanh mL - \frac{h}{mK}\right]\theta_b}{1 + \frac{h}{mK} \tanh mL}$$

$$\therefore Q = -KA \left.\frac{d\theta}{dx}\right|_{x=0}$$

$$Q = -KA \frac{\left[-m \tanh mL - \frac{h}{mK}\right]\theta_b}{1 + \frac{h}{mK} \tanh mL}$$

$$Q = \frac{KAm\left(\tanh mL + \frac{h}{mK}\right)}{1 + \frac{h}{mK} \tanh mL} \qquad m^2 = \frac{hP}{KA}$$

$$Q = \frac{\sqrt{hPKA}\left(\tanh mL + \frac{h}{mK}\right)(T_b - T_\infty)}{\left(1 + \frac{h}{mK} \tanh mL\right)} \qquad m = \sqrt{\frac{hP}{KA}}$$

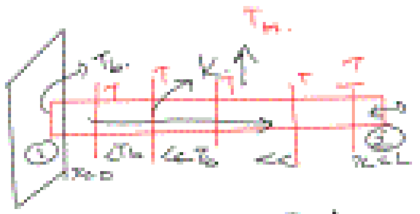
——— x ——— x ———

Fin efficiency:

It is the ratio of the energy transferred through an actual fin to that transferred through an ideal fin.

(R) Ideal fin is one that is made of a perfect (or) infinite conductor material. A perfect conductor has an infinite thermal conductivity so that the entire fin is at the base material temperature.

$$\eta = \frac{q_{actual}}{q_{ideal}} = \frac{hA(T-T_\infty)}{hA_s(T_b-T_\infty)} \qquad \left[\frac{Q_{actual}}{Q_{max}}\right]$$



Between point ① & ② temperature drop is negligible, if K ↑ m.

∴ Heat transfer through any fin can be written as

$$q_{actual} = \eta . h A_{fin}(T_b - T_\infty)$$

For a long fin.

$$\eta_{fin} = \frac{\sqrt{hPkA}\ \tanh mL\ (T_b - T_\infty)}{h(PL)(T_b - T_\infty)}$$

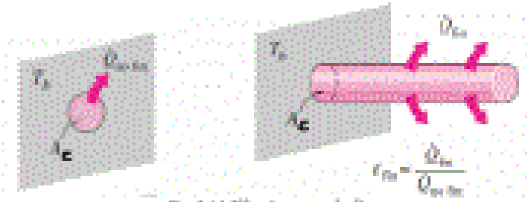
$$\eta_{fin} = \sqrt{\frac{kA}{hP}} \cdot \frac{\tanh mL}{L}$$

$$\eta_{fin} = \frac{\tanh mL}{mL}.$$

Fin Effectiveness: (ϵ)

It is the ratio of fin heat transfer and the heat transfer without the fin.

$$\epsilon = \frac{Q_{fin}}{Q_{no\,fin}} = \frac{Q_{fin}}{hA_c(T_b - T_\infty)}.$$



$$\epsilon = \frac{\sqrt{hPkA_c}\ \tanh(mL)\ (T_b - T_\infty)}{hA_c(T_b - T_\infty)}$$

$$\epsilon = \frac{\sqrt{hPkA_c}\ \tanh mL}{hA_c} = \sqrt{\frac{kP}{hA_c}}\ \tanh mL$$

If the fin is long enough, $mL > 2$, $\tanh mL \to 1$ and hence it can be considered as infinite fin

$$\therefore \quad \epsilon = \sqrt{\frac{kP}{hA_c}} = \sqrt{\frac{k}{h} \cdot \frac{P}{A_c}}$$

Observations:

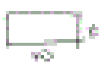
i) To enhance heat transfer ϵ should be greater than unity ($\epsilon > 1$)

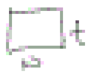
ii) If $\epsilon < 1$, the fin would have no purpose as it would serve as an insulator.

iii) To increase 'ϵ', the fin material should have higher thermal conductivity for a lower heat transfer coefficient 'h'. For an higher heat transfer coefficient 'h', it is not necessary to enhance heat transfer by addition of fins.

$$m = \sqrt{\frac{hP}{kA}}$$

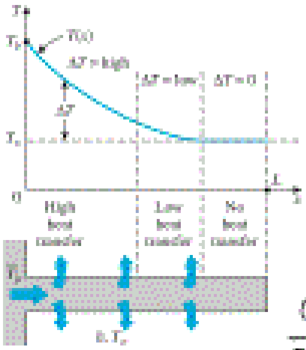
h = ambient condition (h) fixed
k = material (k) fixed.

1)  $A = wt$ $\quad P = 2(w+t)$ $\quad \frac{P}{A} \uparrow$

2)  $A = w^2$ $\quad P = 2(2w) = 4w$

3)  $A = \frac{1}{2} wh$ $\quad P = (w + 2L)$

Proper length of a fin:

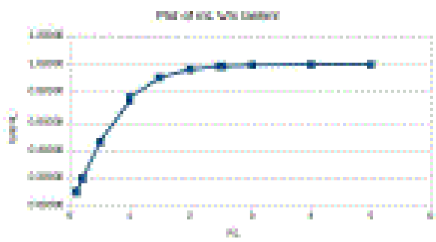


To arrive at the proper length of the fin, compare heat transfer from a fin of finite length to heat transfer from a fin of infinite length under same conditions.

$$\frac{Q_{fin}}{Q_{long\,fin}} = \frac{\sqrt{hPkA}\,(T_b - T_\infty)\tanh mL}{\sqrt{hPkA}\,(T_b - T_\infty)}$$

$$= \tanh mL \longrightarrow (1)$$

mL	$\frac{Q_{fin}}{Q_{long\,fin}} = \tanh mL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000



Observations:

1) $L = 5/m$. [Infinite long]
2) $mL = 5$ to $mL = 2.5$ ↓ 1% Q
3) $mL = 1 \Rightarrow$ 76.2% $Q_{long\,fin}$.
4) $\frac{h\delta}{k_{fin}} << 1$ [0.2]

Extended surfaces:

1. A copper pin fin of 0.25 cm dia, protrudes from a wall at 95^0C into ambient air at 25^0C. The heat transfer is mainly by free convection with heat transfer coefficient value of $10\ W/m^2 - K$. Calculate the heat loss assuming that the fin is infinitely long. For copper, take k=395 W/m-K.

Sol: Given: $d = 0.25cm = 0.25 \times 10^{-2} m$; $T_b = 95°C$ $T_\infty = 25°C$

$h = 10\ W/m^2\text{-}K$. $k = 395\ W/m\text{-}K$.

To find: $Q = ?$

Heat transfer through a long fin is given by.

$$Q = \sqrt{hPKA}\,(T_b - T_\infty) \qquad P = \pi d, \quad A = \pi d^2/4$$

$$= \sqrt{10 \times 7.85 \times 10^{-3} \times 395 \times 4.9 \times 10^{-6}}\ (95 - 25)$$

$Q = 0.865$ W.

2. Aluminium square fins (0.5 mm X 0.5 mm) of 1 cm length are provided on the surface of an electronic semi-conductor device to carry 46 mW of energy generated by the electronic device and the temperature at the surface of the device should not exceed 80^0C. The temperature of the surrounding medium is 40^0C, thermal conductivity of the aluminium is 190 W/m-K and heat transfer coefficient is $12.5\ W/m^2 - K$. Find the number of fins required to carry out the above duty. Neglect the heat loss from the end of the fin.

Sol: Given: $A = 0.5 \times 0.5\ mm^2$. $L = 1cm$; $Q = 46mW$. $T_b = 80°C$

$T_\infty = 40°C$ $k = 190\ W/m\text{-}K$. $h = 12.5\ W/m^2\text{-}K$.

To find: N = number of fins.

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{12.5 \times 2 \times 10^{-3} m}{190 \times 0.25 \times 10^{-6}}} = 22.94.$$

$$\tanh(mL) = \tanh(22.94 \times 1 \times 10^{-2}) = 0.225$$

Single $Q_{fin} = \sqrt{hPKA}\ \tanh mL\ (T_b - T_\infty)$

$$= \sqrt{12.5 \times 2\times10^{-3} \times 190 \times 0.25\times10^{-6}} \times 0.225\ (80-40)$$

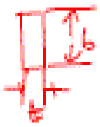
$$Q_{fin} = 9.83\times10^{-3}\ W.$$

$$Q = N \times Q_{fin} \Rightarrow N = \frac{Q}{Q_{fin}} = \frac{46\times10^{-3}}{9.83\times10^{-3}} \simeq 5$$

3. A cylinder 5 cm diameter and 50 cm long is provided with 14 longitudinal straight fins of 1 mm thick and 2.5 mm height. Calculate the heat loss from the cylinder per second if the surface temperature of the cylinder is 200°C

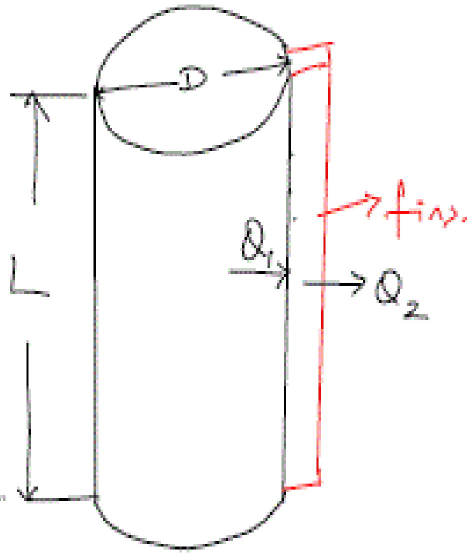
Sol: Given: $D = 5cm$; $L = 50cm$; $N = 14$ fins.

$t = 1mm$; $b = 2.5mm$ $A_f = L \times t$. $P = 2[b+t]$.



$$m = \sqrt{\frac{hP}{KA}}$$

$$A_{cyl} = \pi d L - 14 A_f$$



$$Q_t = Q_{cylinder} + Q_{fins}$$

$$= hA_{cyl}(T_b - T_\infty)$$

$$+ 14\sqrt{hPKA}\,(T_b - T_\infty)\tanh mL$$

$$Q_t =$$

4. To determine the thermal conductivity of a long, solid 2.5 cm diameter rod, one half of the rod was inserted to a furnace while the other half was projecting into air at 27°C. After steady state had been reached, the temperatures at two points 7.6 cm apart were measured and found to be 126°C and 91°C, respectively. The heat transfer coefficient over the surface of the rod exposed to air was estimated to be $22.7 W/m^2 - K$. What is the thermal conductivity of the rod.

Known:- $d = \phi 2.5 \times 10^{-2}$ m; $T_\infty = 27°C$; $T_A = 126°C$, $T_B = 91°C$, $x_{A-B} = 7.6$ cm; $h = 22.7 \; W/m^2\text{-}K$.

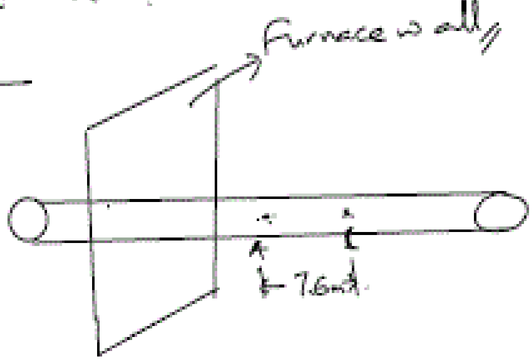
To find:- $k = ?$

Sketch

L = ∞

$T_b \simeq T_A = 126°C$

$T_L \approx T_B = 91°C$



Temp distribution for long fin (Assumption).

$$\frac{(T(L) - T_\infty)}{T_b - T_\infty} = e^{-mx} \qquad x = L = 7.6\,\text{cm}.$$

$$\frac{(91 - 27)}{(126 - 27)} = e^{-m \times 7.6 \times 10^{-2}}$$

$$m = 5.74$$

$$P = \pi d = \pi \times 2.5 \times 10^{-2} = 0.078 \text{ m.}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (2.5 \times 10^{-2})^2}{4} = 4.91 \times 10^{-4} m^2$$

We know, $m^2 = \frac{hP}{kA} \longrightarrow k = \frac{hP}{m^2 A} = \frac{22.7 \times 0.078}{5.74^2 \times 4.94 \times 10^{-4}}$

$$k = 110. \; W/m\text{-}K.$$

5. 30 circular fins of 12 mm diameter and length 500 mm is used to remove heat from the surface maintained at 100^oC to ambient conditions at 25^oC. If the thermal conductivity of the fin material is 250 W/m-K and the corresponding heat transfer coefficient to be $2W/m^2K$. Assuming the fins to be short with convective end. Calculate

   (a) Temperature at a distance of 250 mm from the surface along the fin.

   (b) Total heat transfer through the fins

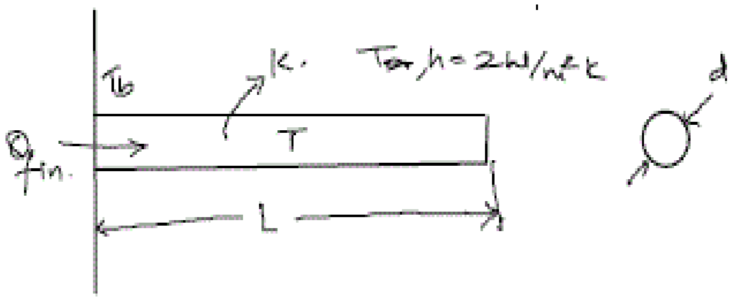
   (c) If the heat transfer coefficient is varient in steps of 10 from a value of 2 W/m^2K to 100 W/m^2K, what would be the change in heat transfer through the fins.

Sol: Given:

$L = 500mm$, $K = 250\ W/mK$; $d = 12mm$, $N = 30$, $h = 2\ W/m^2K$

$T_b = 100^\circ C$; $T_\infty = 25^\circ C$. To find: $T_{x=250mm} = ?$ $Q_{fins} = ?$

$\eta_{fin} = ?$ Q when $h = 2$ to $100\ W/m^2K$ (in steps of 10) = ?



Perimeter $P = \pi d = \pi(12\times10^{-3}) = 37.7\times10^{-3}\ m$.

c/s Area $A = \frac{\pi d^2}{4} = \frac{\pi(12\times10^{-3})^2}{4} = 1.13\times10^{-4}\ m^2$

$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{2\times37.7\times10^{-3}}{250\times1.13\times10^{-4}}} = 1.63$

$mL = 1.63\times500\times10^{-3} = 0.815$.

$\tanh mL = 0.679$ ≈ 1

(i) Temperature distribution:

$$\frac{(T(x) - T_\infty)}{(T_b - T_\infty)} = \frac{\cosh m(L-x) + \frac{h}{mk}\sinh m(L-x)}{\cosh mL + \frac{h}{mk}\sinh mL}$$

(a) $x = 250\,mm$

$$\frac{\left(T(x)\big|_{x=250} - 25\right)}{(100 - 25)} = \frac{\cosh[1.61(0.5-0.25)] + \frac{2}{1.61 \times 250}\sinh[1.61(0.5-0.25)]}{\cosh(1.61 \times 0.5) + \frac{2}{1.61 \times 250}\sinh[1.61 \times 0.5]}$$

$$T(x)\big|_{x=250\,mm} = 59.71 \text{ degree C}$$

(b) $$Q_{fin} = \frac{\sqrt{hPKA}\left(\tanh mL + h/mk\right)(T_b - T_\infty)}{\left(1 + h/mk \cdot \tanh mL\right)}$$

$$Q_{fin} = \frac{\sqrt{2 \times 37.7 \times 10^{-3} \times 250 \times 1.13 \times 10^{-4}}\ \tanh(0.815) + \frac{2}{1.61 \times 250}(100 - 25)}{\left(1 + \frac{2}{1.61 \times 250}\tanh(0.815)\right)}$$

Q_{fin} = 22.92 kW

$Q_{total} = 30 \times Q_{fin} =$ 687.73 kW

(c) Use excel sheet to find the Q_{fin} for h varying from 2 to 100 (steps of 10) and plot Q vs h.