TRANSIENT CONDUCTION: Lumped parameter analysis, Use of Transient temperature charts (Heisler's charts) for transient conduction in slab, long cylinder and sphere. **03 Hours**

Three dimensional unsteady conduction equation

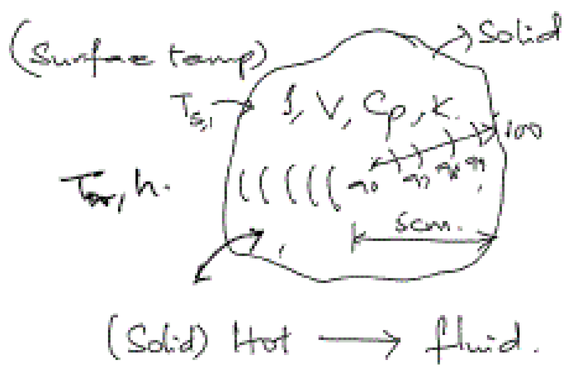
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{K} = \left[\frac{1}{\alpha}\frac{\partial T}{\partial t}\right] \rightarrow (1) \quad \left[\alpha = \frac{K}{\rho C_p}\right]$$

For no-heat generation and considering spacial. variation of temperature only in x-direction,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{dT}{dt} \rightarrow (2)$$

For the solution to equation (2), we need two boundary conditions in x-direction and one initial condition.

Transient conduction (Temperature changes with time)



2 Cases:

1) Small volume:
Temp distribution is uniform

2) Large volume:
Temp distribution is varying.

Lumped thermal capacity analysis:

In this type of analysis we neglect the temperature distribution inside the solid and only deal with the heat transfer between the solid and ambient fluid.

[Temperature inside the solid is constant and is equal to the surface temperature]

Applying energy balance:

$$\cancel{E_{in}}^{0} - E_{out} + \cancel{E_{gen}}^{0} = \frac{\partial E}{\partial t}$$

$$\begin{bmatrix} \text{Heat out of the} \\ \text{object during time } dt \end{bmatrix} = \begin{bmatrix} \text{Decrease in internal} \\ \text{thermal energy of} \\ \text{the object during time } d \end{bmatrix}$$

$$-hA_s[T(t) - T_\infty]\,dt = mC_p\,dT$$

$$-hA_s[T(t) - T_\infty]\,dt = \rho C_p V\,dT \quad [m = \rho V]$$

$$\frac{-hA_s}{\rho C_p V}\,dt = \frac{dT}{(T(t) - T_\infty)}$$

$$\frac{dT}{(T(t) - T_\infty)} = \frac{-hA_s}{\rho C_p V} \rightarrow (1)$$

Let $\theta = T(t) - T_\infty \rightarrow d\theta = dT$

Equation (1) becomes.

$$\frac{d\theta}{dt} = \frac{-hA_s}{\rho C_p V}\theta.$$

$$\frac{d\theta}{\theta} = \frac{-hA_s}{\rho C_p V}$$

$$\frac{d\theta}{\theta} = -Z\,dt \rightarrow (2) \qquad \because Z = \frac{hA_s}{\rho C_p V}\ [t^{-1}]$$

[Reciprocal of Z is called time constant]

t_{ref}.

Integrating equation (2), we get

$$\ln\theta = -Zt + C$$

$$\theta = e^{-(zt+c)}$$

$$\theta = C e^{-zt} \rightarrow (3)$$

Using the initial condition; $t=0$; $T(t) = T_i$

$$T_i - T_\infty = \theta_i = C \rightarrow (4) \quad [@ \text{ initial condition}]$$

Substituting 'c' in (4) we get;

$$\theta = \theta_i e^{-zt} \rightarrow (5)$$

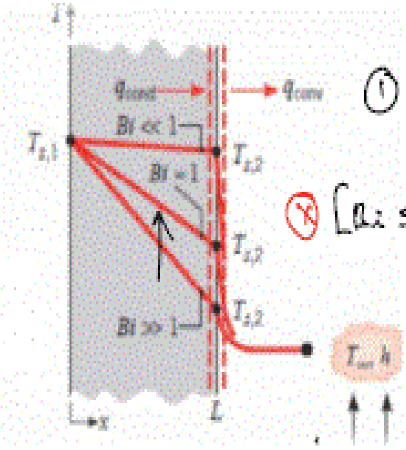
$$\frac{\theta}{\theta_i} = \frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\left[\frac{hA_s}{\rho C_p V}\right]t} \rightarrow (6)$$

Let $L_c = \frac{V}{A_s}$. [Characteristic length].

where, V = Volume

A_s = Surface area.

Criteria for lumped analysis



(1) $$\frac{kA_c}{L}\left(T_{s_1}-T_{s_2}\right) = hA_c\left(T_{s_2}-T_\infty\right)$$

$$\left(T_{s_1}-T_{s_2}\right) = \underbrace{\frac{hL}{k}}_{Bi}\left(T_{s_2}-T_\infty\right)$$

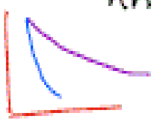
$$\frac{T_{s_1}-T_{s_2}}{T_{s_2}-T_\infty} = Bi \ll 1.$$

[Very small]

[$Bi \leq 0.1$]

(2) $$\frac{hA}{\rho C_p V} t = \frac{t}{t_{ref}} \Rightarrow t_{ref} = \frac{\rho C_p V}{hA} \text{ (Time constant).}$$

$$\frac{\theta}{\theta_i} = e^{-t/t_{ref}}.$$



If t_{ref} large → Longer time required to reach T_∞

If t_{ref} small → Shorter time required to reach T_∞

(3) $$\frac{-hA}{\rho C_p V}t \Rightarrow \frac{-ht}{\rho L_c C_p} * \frac{L_c k}{L_c k} \Rightarrow \frac{-hL_c}{k} * \left[\frac{k}{\rho C_p}\right] \cdot \frac{t}{L_c^2}$$

$$\Rightarrow \underbrace{\frac{-hL_c}{k}}\cdot\underbrace{\frac{\alpha t}{L_c^2}}$$

$Bi = \frac{hL_c}{k}$ [Biot number]

$F_o = \frac{\alpha t}{L_c^2}$ [Fourier number]

So, the equation becomes:

$$\frac{\theta}{\theta_i} = \frac{T-T_\infty}{T_i-T_\infty} = e^{-[B_i F_o]} \rightarrow (7)$$

Heisler and Grober charts:

1. Plane.
2. Cylinder.
3. Sphere

$$\frac{d^2\theta}{dx^2} = \frac{1}{\alpha} \cdot \frac{d\theta}{dt} \qquad \theta = T - T_\infty$$

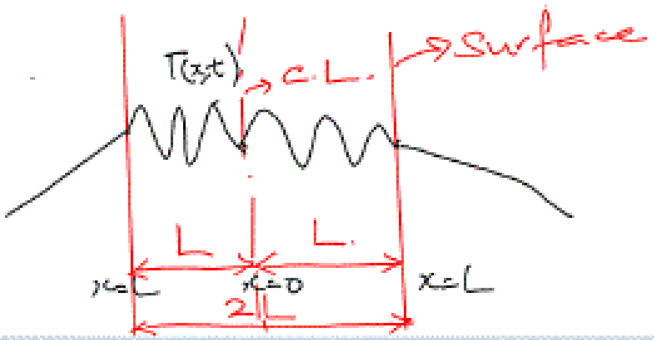
(a) $$\frac{T_x - T_\infty}{T_i - T_\infty} = \text{erf} \frac{x}{2\sqrt{\alpha t}} \qquad Z = \frac{x}{2\sqrt{\alpha t}} \text{ [Tabulated]}$$

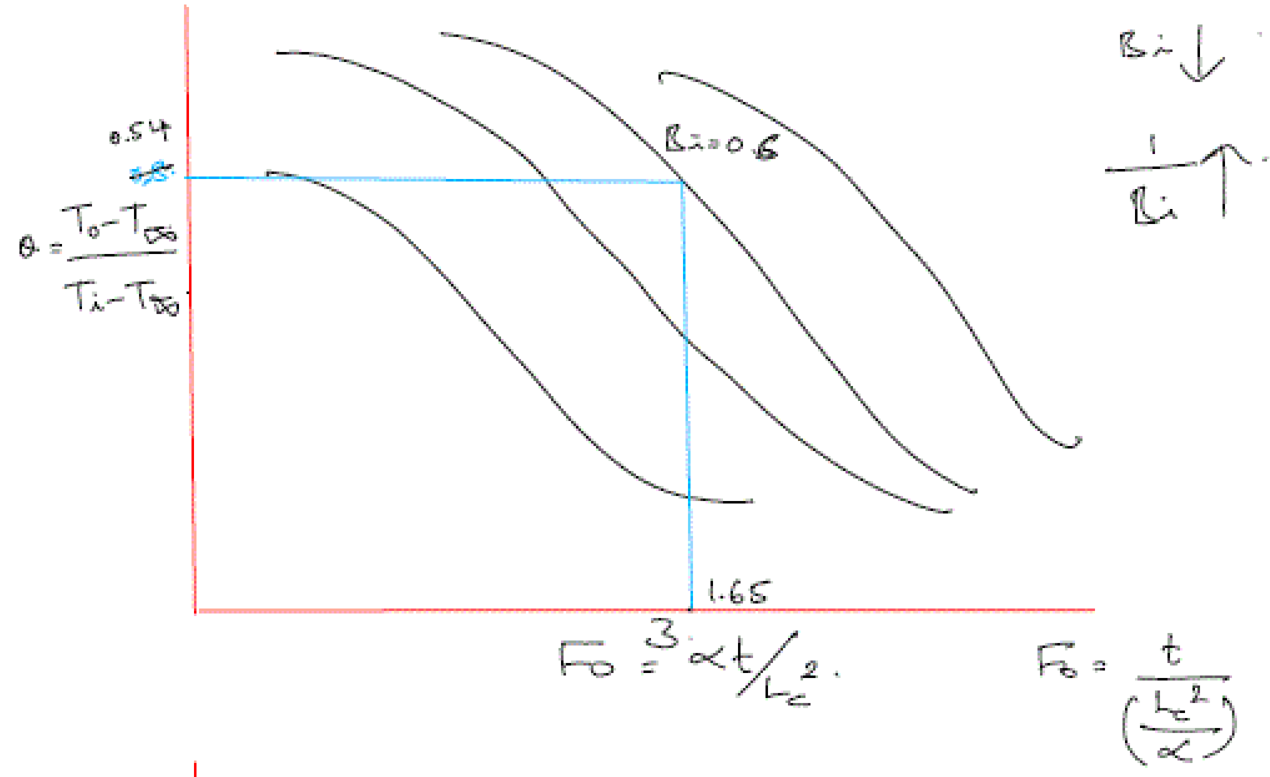
(b) Use charts → Heisler, Grober.

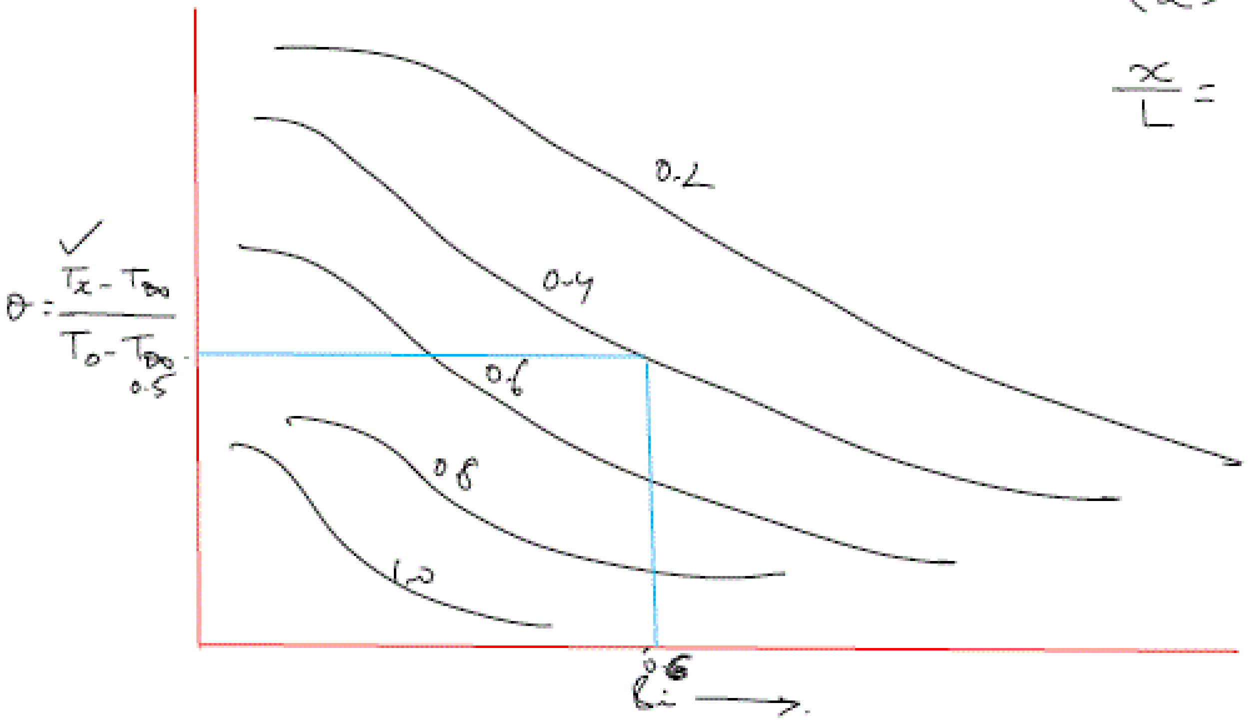
Heisler chart:

1. Main chart → Temperature at the centre line (OR) core.
2. Correction chart → Temperature at any point in 'x' direction.

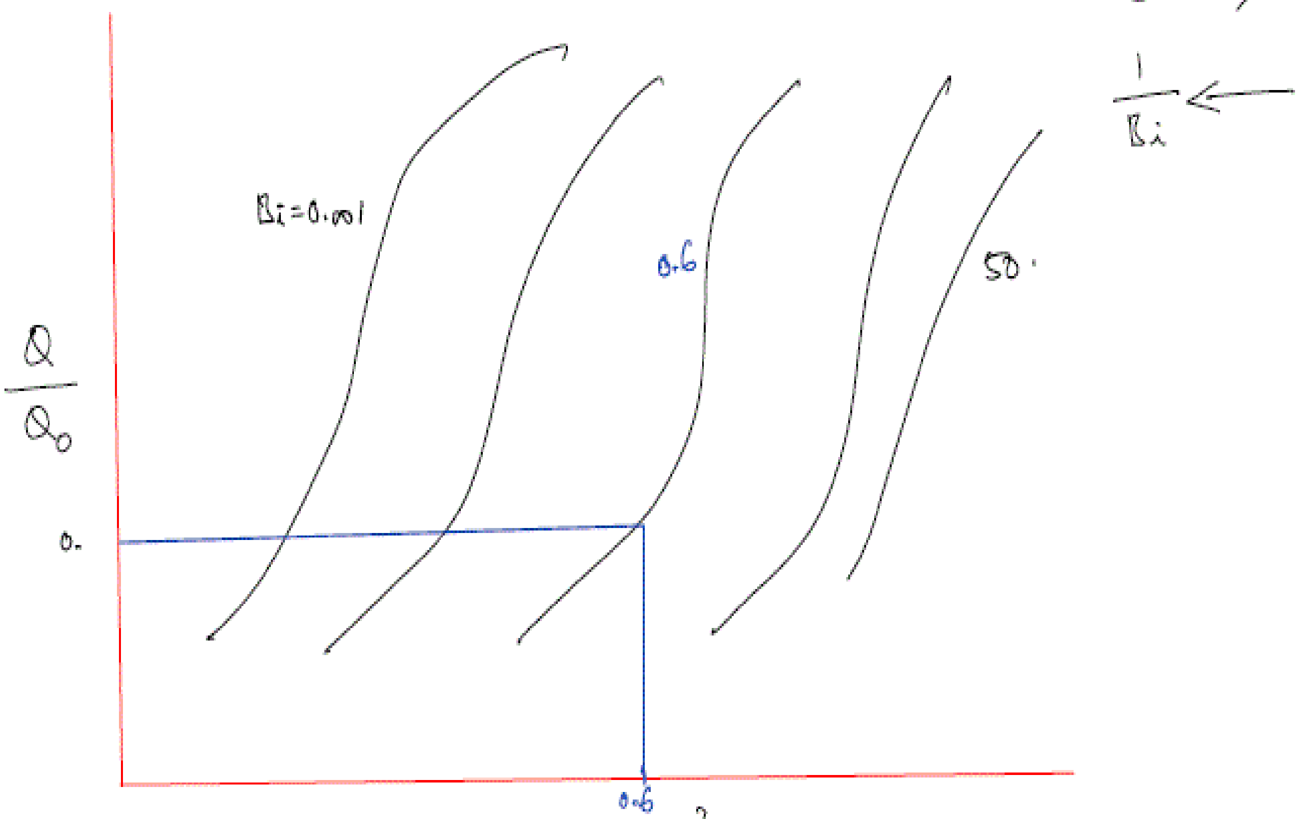
Plane wall




0.54
θ = (To−T∞)/(Ti−T∞)
Bi=0.6
Bi ↓
1/Bi ↑
1.65
Fo = αt/Lc²
Fo = t/(Lc²/α)


x/L =
0.2
0.4
0.6
0.8
1.0
θ = (Tx−T∞)/(To−T∞)
0.5
0.6
Bi →

Grober chart: (Heat flow)



$f(Bi, F_0)$. $\frac{h^2 \alpha t}{k^2}$.

Characteristic length

Cylinder $L_c = \frac{d}{2}$; Sphere $L_c = \frac{r}{3}$; Plane $L_c = \frac{V}{A}$
$= r$.