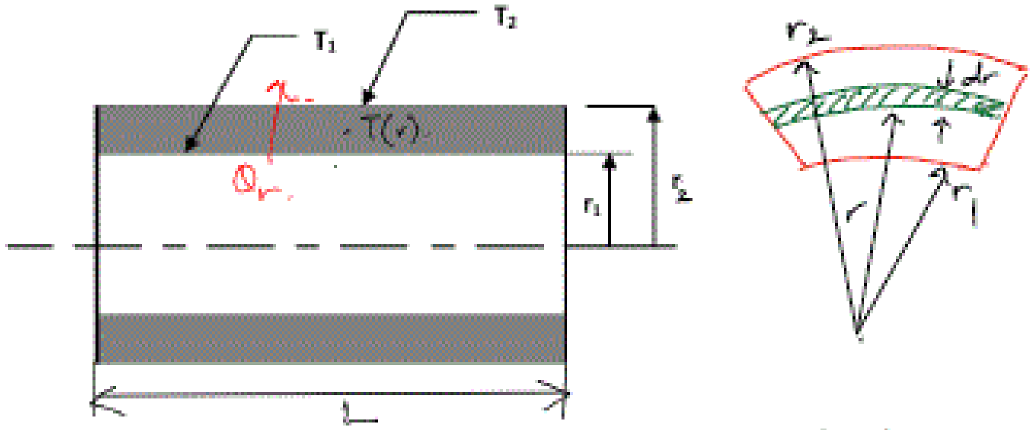
One dimensional conduction equation for cylinder.



In cylindrical coordinates, the conduction eqn for steady state with no heat generation;

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(k\, r\cdot\frac{\partial T}{\partial r}\right)=0$$

For k = constant, above equation can be;

$$\frac{d}{dr}\left(r\cdot\frac{dT}{dr}\right)=0 \longrightarrow (1)$$

Integrating once, we get

$$r\cdot\frac{dT}{dr}=C_1$$

$$\frac{dT}{dr}=\frac{C_1}{r} \longrightarrow (2)$$

Further integration gives;

$$T(r)=C_1 \ln r + C_2 \longrightarrow (3)$$ [Temp distribution].

Boundary Condition → @ $r=r_1$, $T(r)=T_1$
@ $r=r_2$, $T(r)=T_2$.

Substituting these BC's in eqn (3), we get.

$$T_1 = C_1 \ln r_1 + C_2 \rightarrow (4)$$

$$T_2 = C_1 \ln r_2 + C_2 \rightarrow (5)$$

Solve for constants C_1 & C_2.

(5) − (4), $(T_2 - T_1) = C_1 (\ln r_2 - \ln r_1)$

$$\therefore C_1 = \frac{(T_2 - T_1)}{\ln\left(\frac{r_2}{r_1}\right)} \rightarrow (6)$$

Substitute 'C_1' in equation (4) we get.

$$T_1 = \frac{(T_2 - T_1)}{\ln\left(\frac{r_2}{r_1}\right)} \ln r_1 + C_2$$

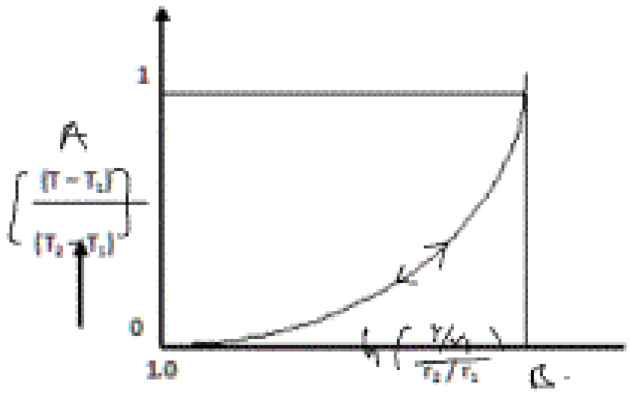
$$\therefore C_2 = T_1 - \frac{(T_2 - T_1)}{\ln\left(\frac{r_2}{r_1}\right)} \ln r_1 \rightarrow (7)$$

Substitute for C_1 & C_2 in equation (3), we get

$$T(r) = \underbrace{\frac{(T_2 - T_1)}{\ln\left(\frac{r_2}{r_1}\right)}}_{C_1} \ln r + \underbrace{T_1 - \frac{(T_2 - T_1)}{\ln\left(\frac{r_2}{r_1}\right)} \ln r_1}_{C_2}$$

$$\frac{T(r) - T_1}{T_2 - T_1} = \frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)} \rightarrow (8)$$

The above equation gives the temperature distribution along the radius of the cylinder;



Expression for the rate of heat transfer.

$$Q_r = -k.A_r.\frac{dT}{dr}$$

$A_r = 2\pi r l = \pi d l$

$A_{r_1} = 2\pi r_1 l = \pi d_1 l$

$A_{r_2} = 2\pi r_2 l = \pi d_2 l$

Hence, $\left.\frac{dT}{dr} = \frac{C_1}{r}\right|_{r=r_1}$

$$\frac{dT}{dr} = \frac{(T_2 - T_1)}{\ln\left(\frac{r_2}{r_1}\right)} * \frac{1}{r_1}$$

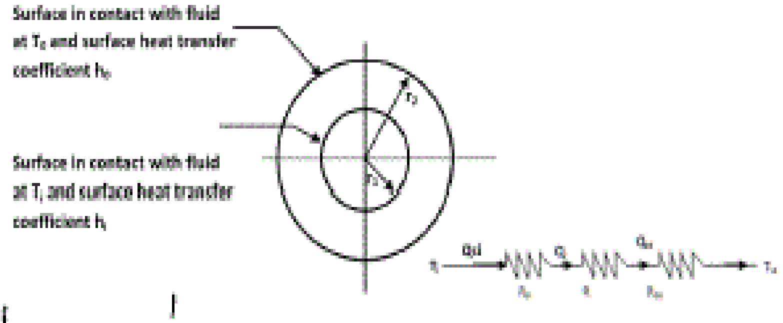
$$Q_r = -k.2\pi r L.\frac{(T_2 - T_1)}{\ln\left(\frac{r_2}{r_1}\right)} * \frac{1}{r_1}$$

$$Q_r = 2\pi k l \frac{(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

$$Q_r = \frac{(T_1 - T_2)}{\left[\frac{\ln(r_2/r_1)}{2\pi k l}\right]}$$

$\rightarrow$ Thermal resistance.

Sphere subjected to convective boundary condition



$$R_{ci} = \frac{1}{h_i A_1} = \frac{1}{h_i\, 4\pi r_1^2}$$

$$R_{co} = \frac{1}{h_o A_2} = \frac{1}{h_o \cdot 4\pi r_2^2}$$

$$R = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k}.$$

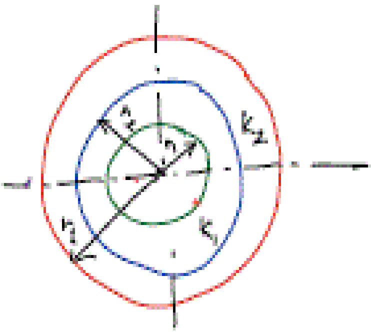
$$R_{total} = R_{ci} + R + R_{co}$$

$$Q = \frac{(T_i - T_o)}{R_{total}}.$$

Composite sphere:

Case 1: No convection BC's.

Thermal resistance circuit:



$$R_1 = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k_1}$$

$$R_2 = \frac{\left(\frac{1}{r_2} - \frac{1}{r_3}\right)}{4\pi k_2}$$

$$R_{total} = R_1 + R_2.$$

$$Q = \frac{T_1 - T_3}{R_{total}}.$$