PART-B

1. From the following distribution of (X,Y) find. (i) $P(X \leq 1)$ (ii) $P(Y \leq 3)$ (iii) $P(X \leq 1, Y \leq 3)$ (iv)$P\left(X \leq \frac{1}{Y} \leq 3\right)$ (v) $P\left(Y \leq \frac{3}{X} \leq 1\right)$ (vi) $P(X + Y \leq 4)$

X \ Y	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

2. The joint probability function (X,Y) is given by $P(x, y) = k(2x + 3y) \quad x = 0,1,2; \; y = 1,2,3$
   (i) Find the marginal distributions.
   (ii) Find the probability distributions of (X+Y)
   (iii) Find all conditional probability distributions.

3. The joint p.d.f of the random variable (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0 < x < 2,\ 0 < y < 1 \\ 0 & otherwise \end{cases}$$

Find

(i) Marginal density function s of X and Y

(ii) Conditional density of X given Y

(iii) $P\left(\frac{1}{4} < X < \frac{\frac{1}{2}}{Y} = \frac{1}{3}\right)$

4. The joint p.d.f of the two dimensional random variable (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{8xy}{9} & : 1 \le x \le y \le 2 \\ 0 & : otherwise \end{cases}$$

Find

(i)Marginal densities of X and Y

(ii) The conditional density functions $f(x/y)$ and $f(y/x)$.

5. If the joint p.d.f of a two dimensional random variable (X,Y) is given by

$$f(x,y) = \begin{cases} x^2 + \frac{xy}{3}: & 0 < x < 1; 0 < y < 2 \\ 0: & otherwise \end{cases}$$

Find (i) $P(X > {}^{1}/_{2})$ (ii) $P(Y > 1)$ (iii) $P(Y < X)$

(iii) $P\left(Y < \frac{1}{2} \middle/ X < \frac{1}{2}\right)$ (v) $P(X + Y \geq 1)$

(vi) find the conditional density functions.
(vii) Check whether the conditional density functions are valid.

6. The joint p.d.f of the random variable (X,Y) is given by
$f(x,y) = kxye^{-(x^2+y^2)}$ $x > 0, y > 0$
(i)Find k (ii) Prove that X and Y are independent.

7. Given
$$f_{XY}(x,y) = \begin{cases} cx(x-y) & ,0 < x < 2, -x < y < x \\ 0 & otherwise \end{cases}$$
(i)Evaluate c
(ii) Find $f_X(x)$
(iii) $f_{\frac{Y}{X}}(y/x)$
(iv) $f_Y(y)$.

8. Two random variables X and Y have the following joint probability density functions

$$f(x,y) = \begin{cases} 2-x-y: 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 \quad : otherwise \end{cases}$$

(i)Find the marginal density functions of X and Y

(ii) Conditional density function

(iii) Var X and Var Y

(iv) Correlation coefficient between X and Y.

9. Given the joint p.d.f of X and Y is

$$f(x,y) = \begin{cases} 8xy: 0 < x < y < 1 \\ 0 \quad : otherwise \end{cases}$$

Find the marginal and conditional p.d.f's X and Y.Are X and Y independent?

10. Let (X,Y) be the two dimensional random variable described by the joint p.d.f

$$f(x,y) = \begin{cases} 8xy: 0 \leq x \leq 1, 0 \leq y \leq x \\ 0 \quad : otherwise \end{cases}$$

Find the Cov(X,Y).

11. The joint p.d.f of the random variable (X,Y) is $f(x,y) = 3(x+y): 0 \leq x \leq 1, 0 \leq y \leq 1, x+y \leq 1.$ Find Cov(X,Y).

12. If X and Y are uncorrelated random variables with variances 16 and 9, find the correlation co-efficient between x+y and x-y.
13. Marks obtained by 10 students in Mathematics(x) and statistics(y) are given below

x:	60	34	40	50	45	40	22	43	42	64
y:	75	32	33	40	45	33	12	30	34	51

Find the two regression lines. Also find y when x=55.

14. In a correlation analysis the equations of the two regression lines are $3x + 12y = 9; and\ 3y + 9x = 46$. Find (i) The value of the correlation coefficient (ii) Mean value of X and Y.
15. Find the correlation coefficient and the equation of the regression lines for the following values of X and Y.

X	1	2	3	4	5
Y	2	5	3	8	7

16. Find the most likely price in City A corresponding to the price of Rs.70 at City B from the following:

	City B	City A

Average Price	65	67
S.D. of Price	2.5	3.5

Correlation coefficient is 0.8.

17. The joint p.d.f of the random variable (X,Y) is given as
$$f(x,y) = \begin{cases} e^{-(x+y)} & ; x > 0, y > 0 \\ 0 & ; otherwise \end{cases}$$
Find the distribution of $\frac{1}{2}(X - Y)$.
18. The independent random variables X and Y follow exponential distribution with parameter $\lambda = 1$. Find the p.d.f of $U = X - Y$.
19. Let X and Y are normally distributed independent random variables with mean 0 and variance σ^2. Find the p.d.f's of $R = \sqrt{x^2 + y^2}$ and $\theta = tan^{-1}\left(\frac{y}{x}\right)$.
20. The joint p.d.f of a two dimensional random variable (X,Y) is given by $f(x,y) = x + y, 0 \le x, y \le 1$. Find the p.d.f of U=XY.

21. If X and Y are independent random variables , with p.d.f $f(x) = e^{-x}, x \geq 0$: $f(y) = e^{-y}, y \geq 0$. Show that $U = \frac{X}{X+Y}$ and $V = X + Y$ are independent.
22. The life time of a particular variety of electric bulbs may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using Central limit theorem find the probability that the average life time of 60 bulbs exceeds 1250 hours.
23. Suppose that orders at a restaurant are i.i.d. random variables with mean $\mu = Rs. 8$ and the standard deviation $\sigma = Rs. 2$. Estimate

(i) The probability that the first 100 customers spend a total of more than Rs.840 i.e., $P(X_1 + X_2 + \cdots + X_n > 840)$

(ii) $P(780 < X_1 + X_2 + \cdots + X_n < 820)$