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INVERSE OF A MATRIX - GAUSS JORDAN METHOD

Let us find the inverse of a non-singular square matrix A of order three. If X is the inverse of A, then AX = I where I is the unit matrix of order 3. Now, we have to find the elements of X.

$$\text{Let } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \text{ and } X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$$

Therefore AX = I reduces to,

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow (1)$$

This equation is equivalent to the three equations given below:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{21} \\ x_{31} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \longrightarrow (2)$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{12} \\ x_{22} \\ x_{32} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \longrightarrow (3)$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{13} \\ x_{23} \\ x_{33} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \longrightarrow (4)$$

From equations (2), (3), (4), we can solve for the vectors $\begin{pmatrix} x_{11} \\ x_{21} \\ x_{31} \end{pmatrix}$, $\begin{pmatrix} x_{12} \\ x_{22} \\ x_{32} \end{pmatrix}$ and $\begin{pmatrix} x_{13} \\ x_{23} \\ x_{33} \end{pmatrix}$ by Gauss-Jordan method. The solution set of each system (2), (3), (4) will be the corresponding column of the inverse matrix X.

Problems :

(1) Find the inverse of the matrix $\begin{pmatrix} 5 & -2 \\ 3 & 4 \end{pmatrix}$ by Gauss-Jordan method.

Solution :

Let $A = \begin{pmatrix} 5 & -2 \\ 3 & 4 \end{pmatrix}$ and $X = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$ be the inverse of A, so that $AX = I$.

Step 1 : The augmented matrix is,

$$\left[\begin{array}{cc|cc} 5 & -2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$$

Step 2 : $R_1 \to R_1/5$

$$\left[\begin{array}{cc|cc} 1 & -2/5 & 1/5 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$$

Step 3 : $R_2 \to R_2 - 3R_1$

$$\left[\begin{array}{cc|cc} 1 & -2/5 & 1/5 & 0 \\ 0 & 26/5 & -3/5 & 1 \end{array}\right]$$

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Step 4 : $R_2 \to R_2 / 26/5$

$$\left[\begin{array}{cc|cc} 1 & -2/5 & 1/5 & 0 \\ 0 & 1 & -3/26 & 5/26 \end{array}\right]$$

Step 5 : $R_1 \to R_1 + (2/5) R_2$

$$\left[\begin{array}{cc|cc} 1 & 0 & 2/13 & 1/13 \\ 0 & 1 & -3/26 & 5/26 \end{array}\right]$$

Hence the inverse of the given matrix is,

$$\begin{bmatrix} 2/13 & 1/13 \\ -3/26 & 5/26 \end{bmatrix} = \frac{1}{26}\begin{bmatrix} 4 & 2 \\ -3 & 5 \end{bmatrix}$$

(2) Find the inverse of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ using Gauss - Jordan method.

Solution :

Let $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ and $X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$

be the inverse of A, so that $AX = I$.

Step 1 : The augmented matrix is,

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right)$$

Step 2 : $R_2 \to R_2 - R_1$, $R_3 \to R_3 + 2R_1$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right)$$

Step 3 : $R_3 \to R_3 + R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right)$$

Step 4 : $R_1 \to 2R_1 + R_2$

$$\left(\begin{array}{ccc|ccc} 2 & 4 & 0 & 1 & 1 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right)$$

Step 5 : $R_2 \to -4R_2 + 6R_3$

$$\left(\begin{array}{ccc|ccc} 2 & 4 & 0 & 1 & 1 & 0 \\ 0 & -8 & 0 & 10 & 2 & 6 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right)$$

Step 6 : $R_1 \to 2R_1 + R_2$

$$\left(\begin{array}{ccc|ccc} 4 & 0 & 0 & 12 & 4 & 6 \\ 0 & -8 & 0 & 10 & 2 & 6 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right)$$

Step 7 : $R_1/4$, $R_2/(-8)$, $R_3/-4$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right)$$

Hence the inverse of the given matrix is

$$\begin{pmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{pmatrix}$$