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GAUSS - JACOBI METHOD

① Solve the following system of equations by using Gauss - Jacobi method.

$$10x + 2y + z = 9 \rightarrow (1)$$
$$x + 10y - z = -22 \rightarrow (2)$$
$$-2x + 3y + 10z = 22 \rightarrow (3)$$

Solution:

Since the given system of equations is diagonally dominant, we write the equations as,

$$x = \frac{1}{10}(9 - 2y - z) \rightarrow (4)$$
$$y = \frac{1}{10}(-22 - x + z) \rightarrow (5)$$
$$z = \frac{1}{10}(22 + 2x - 3y) \rightarrow (6)$$

First iteration:

Putting $x = 0$, $y = 0$ & $z = 0$ in (4), (5) & (6)

$$x = \frac{9}{10} = 0.9$$
$$y = \frac{-22}{10} = -2.2$$
$$z = \frac{22}{10} = 2.2$$

Second iteration:

Putting $x = 0.9$, $y = -2.2$, $z = 2.2$ in (4), (5) & (6),

$$x = \frac{1}{10}(9 + 2 \times 2.2 - 2.2) = 1.12$$
$$y = \frac{1}{10}(-22 - 0.9 + 2.2) = -2.07$$
$$z = \frac{1}{10}(22 + 2 \times 0.9 + 3 \times 2.2) = 3.04$$

Third iteration:

$$x = \frac{1}{10}(9 + 2 \times 2.07 - 3.04) = 1.01$$

$$y = \frac{1}{10}(-22 - 1.12 + 3.04) = -2.008$$

$$z = \frac{1}{10}(22 + 2 \times 1.12 + 3 \times 2.07) = 3.045$$

Fourth iteration:

$$x = \frac{1}{10}(9 + 2 \times 2.008 - 3.045) = 0.9971$$

$$y = \frac{1}{10}(-22 - 1.01 + 3.045) = -1.9965$$

$$z = \frac{1}{10}(22 + 2 \times 1.01 + 3 \times 2.008) = 3.0044$$

Fifth iteration:

$$x = \frac{1}{10}(9 + 2 \times 1.9965 - 3.0044) = 0.9989$$

$$y = \frac{1}{10}(-22 - 0.9971 + 3.0044) = -1.9993$$

$$z = \frac{1}{10}(22 + 2 \times 0.9971 + 3 \times 1.9965) = 2.9984$$

Sixth iteration:

$$x = \frac{1}{10}(9 + 2 \times 1.9993 - 2.9984) = 1$$

$$y = \frac{1}{10}(-22 - 0.9989 + 2.9984) = \overset{-2}{\cancel{1.9995}}$$

$$z = \frac{1}{10}(22 + 2 \times 0.9989 + 3 \times 1.9993) = 2.9996$$

Seventh iteration:

$$x = \frac{1}{10}(9 + 4 - 2.9996) = 1$$

$$y = \frac{1}{10}(-22 - 1 + 2.9996) = -2$$

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$$z = \frac{1}{10}(22 + 2 + 6) = 3$$

$\therefore$ The required solution is

$$\boxed{x = 1,\ y = -2,\ z = 3}$$

(2) Solve the following system of equations, correct to 3 decimal places, using Gauss–Jacobi method.

$$-x + y + 10z = 35.61$$
$$x + 10y + z = 20.08$$
$$10x + y - z = 11.19$$

Solution:

Since the given system of equations is not diagonally dominant, we rewrite the equations as,

$$10x + y - z = 11.19 \longrightarrow (1)$$
$$x + 10y + z = 20.08 \longrightarrow (2)$$
$$-x + y + 10z = 35.61 \longrightarrow (3)$$

Hence we have,

$$x = \frac{1}{10}(11.19 - y + z) \longrightarrow (4)$$
$$y = \frac{1}{10}(20.08 - x - z) \longrightarrow (5)$$
$$z = \frac{1}{10}(35.61 + x - y) \longrightarrow (6)$$

First iteration:

Putting $x = 0$, $y = 0$, $z = 0$ in (4), (5) & (6),

$$x = \frac{11.19}{10} = 1.119$$

$$y = \frac{20.08}{10} = 2.008$$

$$z = \frac{35.61}{10} = 3.561$$

Second iteration:

$$x = \frac{1}{10}(11.19 - 2.008 + 3.561) = 1.274$$

$$y = \frac{1}{10}(20.08 - 1.119 - 3.561) = 1.54$$

$$z = \frac{1}{10}(35.61 + 1.119 - 2.008) = 3.472$$

Third iteration:

$$x = \frac{1}{10}(11.19 - 1.54 + 3.472) = 1.312$$

$$y = \frac{1}{10}(20.08 - 1.274 - 3.472) = 1.533$$

$$z = \frac{1}{10}(35.61 + 1.274 - 1.54) = 3.534$$

Fourth iteration:

$$x = \frac{1}{10}(11.19 - 1.533 + 3.534) = 1.319$$

$$y = \frac{1}{10}(20.08 - 1.312 \cancel{+} - 3.534) = 1.523$$

$$z = \frac{1}{10}(35.61 + 1.312 - 1.533) = 3.539$$

Fifth iteration:

$$x = \frac{1}{10}(11.19 - 1.523 + 3.539) = 1.321$$

$$y = \frac{1}{10}(20.08 - 1.319 - 3.539) = 1.522$$

$$z = \frac{1}{10}(35.61 + 1.319 - 1.523) = 3.539$$

Hence

$$x = 1.321,\ y = 1.522,\ z = 3.539$$