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Test 2:

Test of significance for the difference between two Population means when population standard deviations are not known:

Let \overline{x}_1 and \overline{x}_2 are the means of two independent samples of sizes n, and n₂ from a normal population with mean μ_1 , and μ_2 and standard deviation -s, and s₂.

We want to test whether the mean μ_i and μ_2 of the two populations are equal or not Under H_0 : $\mu_i = \mu_2$ the test-Statistic is defined as,

$$t = \overline{x_1 - x_2}$$

$$S \sqrt{\frac{1}{n_1 + 1}}$$

where $S^2 = n_1 s_1^2 + n_2 s_2^2$ $n_1 + n_2 - 2$

with $v = n_1 + n_2 - 2$ degrees of freedom.

Note :

Suppose the Samples Sizes are equal. i.e., $n_1 = n_2 = n$. Then we have n pair of values. Further we assume that the n pairs are independent. Then





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the test-Statistic will be,

$$t = \overline{x_1 - \overline{x_2}}$$

$$\int \frac{\overline{s_1^2 + \overline{s_2^2}}}{n - 1}$$

with V = n + n - 2 = 2n - 2 degrees of freedom.



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Problems :

(1) Two salesmen A and B are working in a Certain district. From a sample Survey conducted by the Head office, the following results were obtained. State whether there is any significant difference in the average sales between the two salesmen:

	A	В
Number of Sales	20	18
Average Sales (in Rs.)	170	205
Standard Deviation (in Rs.)	20	25

Solution :

Given: $n_1 = 20$, $n_2 = 18$ $\overline{\chi}_1 = 170$, $\overline{\chi}_2 = 205$ $\sigma_1 = 20$, $\sigma_2 = 25$.

Null hypothesis : H_0 : There is no significant difference in the average sales of the two sales men

i.e.,
$$H_0: \mu_1 = \mu_2$$

Alternative hypothesis : $H_1 : \mu_1 \neq \mu_2$

Test-Statistic :

$$t = \frac{\overline{\chi_1 - \chi_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

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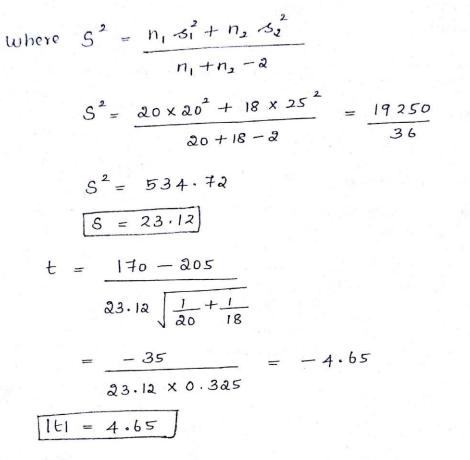


Table value :

At $\alpha = 1$ / Los , $V = n_1 + n_2 - 2 = 36$ d.o.f, the table value of t is given by,

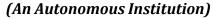
 $t_{\alpha} = 2.58$

Decision :

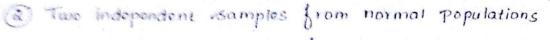
Since ItI > ta, Ho is rejected.

Hence the two salesmen differ Significantly with regard to their average sales





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with equal variance gave the following :

Sample	Size	Mean	S.D
1	16	23.4	2.5
2	12	24.9	2.8

Is the difference between the means significant ? Solution:

Griven:
$$n_1 = 16$$
, $n_2 = 12$
 $\overline{x_1} = 23.4$, $\overline{x_2} = 24.9$
 $s_1 = 2.5$, $s_2 = 2.8$

Null hypothesis: H_0 : There is no significant difference between the means i.e., H_0 : $\mu_1 = \mu_2$

Alternative hypothesis : $H_1 = \mu_1 \neq \mu_2$ (Two-tailed test)

Test-Statistic :

$$t = \frac{\overline{x_{1} - \overline{x_{2}}}}{S\sqrt{\frac{1}{n_{1}} + \frac{1}{n_{2}}}}$$
where $S^{2} = \frac{n_{1} - s_{1}^{2} + n_{2} - s_{2}^{2}}{n_{1} + n_{2} - 2}$

$$S^{2} = \frac{16 \times 2.5^{2} + 12 \times 2.8}{16 + 12 - 2}$$

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$$S^{2} = 7.465$$

$$S = 2.732$$

$$t = 23.4 - 24.9$$

$$a.732 \sqrt{\frac{1}{16} + \frac{1}{12}}$$

$$= -1.5$$

$$1.0433$$

$$\boxed{1} = 1.4378$$

Table value :

At x = 5 % Los, $V = n_1 + n_2 - 2 = 16 + 12 - 2$ V = 26 d.o.f, the table value of t is given by,

Decision :

Since $|t| \ge t_{\alpha}$, H_{o} is accepted. ... The difference is not significant.

A group of 10 Pats fed on diet A and another
group of 8 rats on fed on diet B, Recorded
the following increase in weight (gms)
Diet A: 5, 6, 8, 1, 12, 4, 3, 9, 6, 10
Diet B: 2, 3, 6, 8, 10, 1, 2, 8





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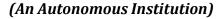
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(115) Does it show superiority of diet A over diet B? Solution : Griven: $n_1 = 10$ $n_{2} = 8$ Null hypothesis : Ho : There is no significant difference in increase of weights . i.e., $H_0: \mu_1 = \mu_2$ Alternative hypothesis: H_1 : $\mu_1 = \mu_2$ (Right tailed test) Calculation of sample means and Sample S. D's: $\overline{\chi}_1 = \underline{\xi \chi_1} = \underline{64} = 6.4$ $\overline{\chi}_2 = \underline{\lesssim} \chi_2 = \underline{40} = 5$ $5_{1}^{2} = \frac{5\chi_{1}^{2}}{n} - \left(\frac{5\chi_{1}}{n}\right)^{2} = \frac{512}{10} - \left(6\cdot4\right)^{2} = 10.24$ $S_{2}^{2} = \frac{\leq \chi_{2}^{2}}{n} - \left(\frac{\leq \chi_{2}}{n}\right)^{2} = \frac{282}{n} - (5)^{2} = 10.25$ Test - Statistic : $t = \overline{\chi_1} - \overline{\chi_2}$ $S \boxed{\frac{1}{n} + \frac{1}{n}}$ Where $S^2 = n_1 s_1^2 + h_2 s_2^2$ $n_{1} + n_{2} - 2$

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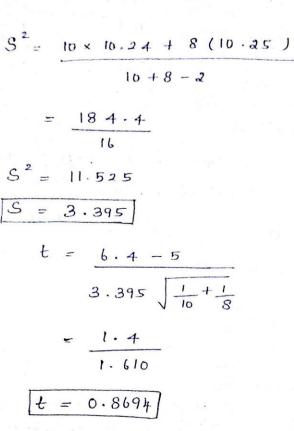


Table value :

At $\alpha = 5$ / Los, $V - n_1 + n_2 - 2 = 16$ d.o.f, the table value of t is given by,

$$t_{\chi} = 1.75$$

Decision :

Since $t \ge t_{\alpha}$, H_{0} is accepted. Hence we Cannot Conclude that diet A is superior to diet B.

