



27. Solve $\frac{d^2y}{dx^2} + y = \csc x$ using method of variation of parameters.

Soln.

Given $(D^2 + 1)y = \csc x$.

AE

$$m^2 + 1 = 0$$

$$m^2 = -1$$

$$m = \pm i$$

$$CF = C_1 \cos x + C_2 \sin x$$

$$\text{Here } \begin{cases} f_1 = \cos x & f_2 = \sin x \\ f_1' = -\sin x & f_2' = \cos x \end{cases}$$

Now $f_1 f_2' - f_1' f_2 = w$

$$w = \cos x (\cos x) + \sin x \cdot \sin x$$

$$= \cos^2 x + \sin^2 x$$

$$w = 1$$

$$PI = Pf_1 + Qf_2$$

$$P = - \int \frac{f_2 x}{w} dx$$

$$= - \int \frac{\sin x \cdot \csc x}{1} dx$$

$$= - \int \sin x \times \frac{1}{\sin x} dx$$

$$= - \int dx$$

$$P = -x$$

$$Q = \int \frac{f_1 x}{w} dx$$

$$= \int \frac{\cos x \cdot \csc x}{1} dx$$

$$= \int \cos x \times \frac{1}{\sin x} dx = \int \cot x dx$$



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$$= \log(\sin x)$$



\therefore PI = $-\alpha (\cos x) + \log (\sin x) \sin x$
The general soln. is,

$$y = CF + PI$$

$$= C_1 \cos x + C_2 \sin x - \alpha \cos x + \sin x \log (\sin x)$$

3]. Solve $\frac{d^2 y}{dx^2} + y = \cot x$ using method of variation of parameters.

Soln.

Given $\Rightarrow (D^2 + 1)y = \cot x$

AE

$$m^2 + 1 = 0$$

$$m^2 = -1$$

$$m = \pm i$$

$$CF = C_1 \cos x + C_2 \sin x$$

Here $f_1 = \cos x \quad | \quad f_2 = \sin x$

$$f_1' = -\sin x \quad | \quad f_2' = \cos x$$

Now,

$$w = f_1 f_2' - f_1' f_2$$

$$= 1$$

$$PI = Pf_1 + Qf_2$$

Here $P = -\int \frac{f_2 x}{w} dx$

$$= -\int \frac{\sin x \cot x}{1} dx$$

$$= -\int \sin x \frac{\cos x}{\sin x} dx$$

$$= -\int \cos x dx$$

$$P = -\sin x$$

and $Q = \int \frac{f_1 x}{w} dx = \int \frac{\cos x \cot x}{1} dx$

$$= \int \cos x \frac{\cos x}{\sin x} dx$$





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UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Method of variation of parameters

$$= \int \frac{\cos^2 x}{\sin x} dx$$

$$= \int \frac{1 - \sin^2 x}{\sin x} dx$$

$$= \int [\csc x - \sin x] dx$$

$$= \int \csc x dx - \int \sin x dx$$

$$= -\log |\csc x + \cot x| + \cos x$$

$$\therefore \text{PI} = -\sin x \cos x + \left[\log (\csc x + \cot x) + \cos x \right] \sin x$$

The general soln. is,

$$y = CF + PI$$

$$= C_1 \cos x + C_2 \sin x - \sin x \cos x + \left[\log (\csc x + \cot x) + \cos x \right] \sin x$$

$$= C_1 \cos x + C_2 \sin x + \log (\csc x + \cot x) \sin x.$$

4]. Solve $(D^2 + a^2)y = \sec ax$ using method of variation of parameters.

Soln.

Given $(D^2 + a^2)y = \sec ax$

AE

$$m^2 + a^2 = 0$$

$$m^2 = -a^2$$

$$m = \pm ai$$

$$CF = C_1 \cos ax + C_2 \sin ax$$

$$\text{Here } f_1 = \cos ax \quad \left| \quad f_2 = \sin ax \right.$$

$$f_1' = -a \sin ax \quad \left| \quad f_2' = a \cos ax \right.$$

$$\omega = f_1 f_2' - f_1' f_2$$

$$= \cos ax (a \cos ax) + a \sin ax \sin ax$$

$$= a \cos^2 ax + a \sin^2 ax$$

$$= a [\cos^2 ax + \sin^2 ax] = a(1) = a$$

$$\Rightarrow \omega = a$$



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$$PI = Pf_1 + Qf_2$$

$$P = - \int \frac{f_2 x}{\omega} dx$$

$$= - \int \frac{\sin ax \sec ax}{a} dx$$

$$= - \frac{1}{a} \int \sin ax \frac{1}{\cos ax} dx = - \frac{1}{a} \int \tan ax dx$$

$$= + \frac{1}{a} \frac{\log(\sec ax)}{a} \quad \int \tan ax = \frac{\log(\sec ax)}{a}$$

$$P = - \frac{1}{a^2} \log(\sec ax)$$

$$Q = \int \frac{f_1 x}{\omega} dx$$

$$= \int \frac{\cos ax \sec ax}{a} dx$$

$$= \frac{1}{a} \int \cos ax \frac{1}{\cos ax} dx$$

$$= \frac{1}{a} \int dx$$

$$Q = \frac{x}{a}$$

$$PI = - \frac{1}{a^2} \log(\sec ax) \cos ax + \frac{x}{a} \sin ax$$

The general soln. is,

$$y = CF + PI$$

$$= C_1 \cos ax + C_2 \sin ax + \frac{1}{a^2} \log(\sec ax) \cos ax$$

$$+ \frac{x}{a} \sin ax.$$

