



## UNIT-I VECTOR CALCULUS

## SOLENOIDAL AND IRRROTATIONAL

Solenoidal & Irrotational vector:

Solenoidal vector :  $\nabla \cdot \vec{F} = 0$

Irrotational vector :  $\nabla \times \vec{F} = \vec{0}$

Problems

1] Find 'a' such that  $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$  is solenoidal.

Soln.

Given  $\vec{F} = (3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$   
and  $\nabla \cdot \vec{F} = 0$

$$\frac{\partial}{\partial x} (3x - 2y + z) + \frac{\partial}{\partial y} (4x + ay - z) + \frac{\partial}{\partial z} (x - y + 2z) = 0$$

$$3 + a + 2 = 0$$

$$\boxed{a = -5}$$

2] Show that  $\vec{F} = x\vec{i} + x\vec{j} + y\vec{k}$  is solenoidal.

Soln.

Given  $\vec{F} = x\vec{i} + x\vec{j} + y\vec{k}$

To prove  $\nabla \cdot \vec{F} = 0$

$$\nabla \cdot \vec{F} = \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (x\vec{i} + x\vec{j} + y\vec{k}) =$$

$$= \frac{\partial}{\partial x} (x) + \frac{\partial}{\partial y} (x) + \frac{\partial}{\partial z} (y)$$

$$\nabla \cdot \vec{F} = 0$$

$\therefore \vec{F}$  is solenoidal

3] Show that  $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$  is irrotational.

Soln.

Given  $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$

To prove  $\nabla \times \vec{F} = \vec{0}$



Scanned with CamScanner



Now,

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix}$$

$$= \vec{i} \left[ \frac{\partial}{\partial y} (zy) - \frac{\partial}{\partial z} (zx) \right] - \vec{j} \left[ \frac{\partial}{\partial x} (zy) - \frac{\partial}{\partial z} (yx) \right] + \vec{k} \left[ \frac{\partial}{\partial x} (zx) - \frac{\partial}{\partial y} (yz) \right]$$

$$= \vec{i} [z - z] - \vec{j} [y - y] + \vec{k} [z - z]$$

$$= 0\vec{i} + 0\vec{j} + 0\vec{k}$$

$$= \vec{0}$$

∴  $\vec{F}$  is irrotational.

AJ. If  $\vec{A}$  and  $\vec{B}$  are irrotational. Prove that  $\vec{A} \times \vec{B}$  is solenoidal.

Soln.

Given  $\vec{A}$  &  $\vec{B}$  are irrotational.

i.e.,  $\nabla \times \vec{A} = \vec{0}$  and  $\nabla \times \vec{B} = \vec{0}$

WKT  $\nabla \cdot (\vec{A} \times \vec{B}) = (\nabla \times \vec{A}) \cdot \vec{B} - (\nabla \times \vec{B}) \cdot \vec{A}$

$$= \vec{0} \cdot \vec{B} - \vec{0} \cdot \vec{A}$$

$$= 0 - 0$$

$$= 0$$

Hence  $\vec{A} \times \vec{B}$  is solenoidal.

QJ. Find the values of a, b, c so that the vector  $\vec{F} = (x+y+az)\vec{i} + (bx+2y-z)\vec{j} + (-x+cy+2z)\vec{k}$  may be irrotational.

Soln.

Given  $\vec{F}$  is irrotational.

i.e.,  $\nabla \times \vec{F} = \vec{0}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y+az & bx+2y-z & -x+cy+2z \end{vmatrix} = \vec{0}$$





# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-I VECTOR CALCULUS

SOLENOIDAL AND IRROTATIONAL

$\vec{i}(c+1) - \vec{j}(-1-a) + \vec{k}(b-1) = 0\vec{i} + 0\vec{j} + 0\vec{k}$   
 Equating the coefficients of like terms,

$$\begin{aligned} c+1=0, & \quad 1+a=0, & \quad b-1=0 \\ c=-1 & \quad a=-1 & \quad b=1 \end{aligned}$$

6]. Show that  $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$  is irrotational and hence find its scalar potential.

Soln.

Given  $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$

Now

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 + 2xz^2 & 2xy - z & 2x^2z - y + 2z \end{vmatrix}$$

$$= \vec{i}[-1+1] - \vec{j}[4xz-4xz] + \vec{k}[2y-2y]$$

$$= 0\vec{i} - 0\vec{j} + 0\vec{k}$$

$$\nabla \times \vec{F} = \vec{0}$$

Hence  $\vec{F}$  is irrotational.

$$\Rightarrow \vec{F} = \nabla \phi$$

$$(y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$$

$$= \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$$

Equating the coefficients of  $\vec{i}$ ,  $\vec{j}$  &  $\vec{k}$ , we get

$$\frac{\partial \phi}{\partial x} = y^2 + 2xz^2 \quad \left| \quad \frac{\partial \phi}{\partial y} = 2xy - z \quad \left| \quad \frac{\partial \phi}{\partial z} = 2x^2z - y + 2z \right. \right.$$

Integrating partially w.r. to  $x, y, z$ ,

$$\phi = xy^2 + x^2z^2 + f_1(y, z) \rightarrow (1)$$

$$\phi = xy^2 - yz + f_2(x, z) \rightarrow (2)$$

$$\phi = x^2z^2 - yz + z^2 + f_3(x, y) \rightarrow (3)$$

Comparing (1), (2), and (3), we get

$$\phi = xy^2 - yz + x^2z^2 + z^2 + c, \text{ where } c \text{ is the arbitrary constant}$$



Scanned with CamScanner