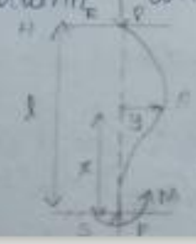
SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

COIMBATORE-641 035, TAMIL NADU

2) column with one end fixed other end hinged



consider a column R.S of length l and uniform cross sectional area a carrying a critical load P at both ends R, S

* the column is fixed at end S and hinged at end R
* due to application of load P the column will deflection as shown in fig.
* there will be horizontal reaction H at R as shown in fig

Moment at a point A = $-Py + H(l-x)$ $\rightarrow (1)$

moment due to critical load + moment due to horizontal reaction at 'R'

i.e. the deflection distance

$$M = -Py + H(l-x)$$

$$M = EI \frac{d^2y}{dx^2} \rightarrow (2)$$

equating (1) & (2)

$$EI \frac{d^2y}{dx^2} = -Py + H(l-x)$$

÷ EI on above eqn

$$\frac{d^2y}{dx^2} = \frac{-P}{EI} y + \frac{H}{EI}(l-x)$$

$\div$ EI in above eqn

$$\frac{d^2y}{dx^2} = \frac{-P}{EI} y + \frac{H}{EI} (l-x)$$

$\times$ & $\div$ by P/P on RHS

$$\frac{d^2y}{dx^2} + P/EI\, y = H/EI\, (l-x) \times P/P$$

$$\frac{d^2y}{dx^2} + P/EI\, y = P/EI \times H/P\, (l-x)$$

The general eqn is

$$y = A \cos(x\sqrt{P/EI}) + B \sin(x\sqrt{P/EI}) + H/P\, (l-x)$$

At a point S the values of x & y are

$x=0;\ y=0$ sub the values in general eqn.

$$0 = A \cos(0\sqrt{P/EI}) + B \sin(0\sqrt{P/EI}) + H/P\,(l-0)$$

$$0 = A \cos(0) + H/P(l)$$

$$0 = A + H/P\,(l)$$

$$A = -H/P\,(l)$$

Diff the general eqn w.r.to x

$$\frac{dy}{dx} = -A \sin(x\sqrt{P/EI}) \times \sqrt{P/EI} + B\cos(x\sqrt{P/EI})(\sqrt{P/EI}) - H/P$$

$$\frac{dy}{dx} = -\sqrt{P/EI}\, A \sin(x\sqrt{P/EI}) + \sqrt{P/EI}\, B\cos(x\sqrt{P/EI}) - H/P$$

$$0 = \sqrt{P/EI} \times H/P\, l \sin(0\sqrt{P/EI}) + \sqrt{P/EI}\, B \cos(0\sqrt{P/EI}) - H/P$$

$$0 = \sqrt{P/EI}\, B - H/P$$

$$H/P = B\sqrt{P/EI}$$

$$B = H/P\sqrt{EI/P}$$

sub A & b value in general eqn

$$y = -H/P\, l \cos(x\sqrt{P/EI}) + H/P\sqrt{EI/P} \sin(x\sqrt{P/EI}) + H/P(l-x)$$

at a point R $x = l;\ y=0$ sub in above eqn

$$0 = -H/P\, l \cos(l\sqrt{P/EI}) + H/P\sqrt{EI/P} \sin(l\sqrt{P/EI}) + H/P(l-l)$$

$$0 = -H/P\, l \cos(l\sqrt{P/EI}) + H/P\sqrt{EI/P} \sin(l\sqrt{P/EI})$$

$$H/P\, l \cos(l\sqrt{P/EI}) = H/P\sqrt{EI/P} \sin(l\sqrt{P/EI})$$

$$l \cos\left(l\sqrt{P/EI}\right) = \sqrt{\frac{EI}{P}} \sin\left(l\sqrt{P/EI}\right)$$

$$\frac{l}{\sqrt{EI/P}} = \frac{\sin}{\cos}\left(l\sqrt{P/EI}\right)$$

$$l \times \sqrt{P/EI} = \tan\left(l\sqrt{P/EI}\right)$$

$$l \times \sqrt{P/EI} = 4.5 \text{ radia}$$

$$\sqrt{P/EI} = \frac{4.5}{l}$$

squaring on both side

$$P/EI = \frac{20.25}{l^2}$$

$$\boxed{P = \frac{2\pi^2 EI}{l^2}}$$