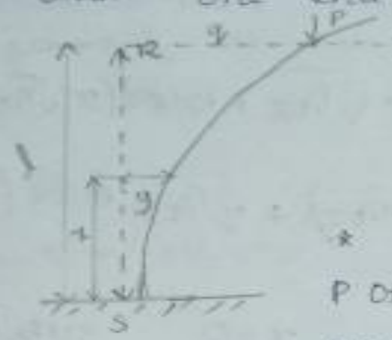
column with one end fixed other end free



* let due to crippling load P of the column just buckle.

* let a be the deflection of the top end.

Taking moment = load × distance

$$M = P \times (a-y)$$

$$M = Pa - Py$$

$$M = EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = Pa - Py$$

$\div$ EI

$$\frac{d^2y}{dx^2} = \frac{P}{EI} a - \frac{P}{EI} y$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{P}{EI} a$$

The solution of above differential eqn is

$$y = A \cos\left(x\sqrt{P/EI}\right) + B \sin\left(x\sqrt{P/EI}\right) + a$$

Applying the end condition at point 'S'

$x = 0$; $y = 0$

$$y = 0 = \underset{\text{original}}{A\cos\left(0\sqrt{P/EI}\right)} + B\sin\left(x\sqrt{P/EI}\right) + a \longrightarrow (1)$$

$$= A\cos 0 + B(0) + a \Rightarrow 0 = A + 0 + a \Rightarrow \boxed{A = -a}$$

differentiate the general eqn(1) w.r.to 'x'
the slope at any section is given by

$$\frac{dy}{dx} = -A \sin\left(x\sqrt{P/EI}\right) \times \sqrt{P/EI} + B\cos\left(x\sqrt{P/EI}\right) \times \sqrt{P/EI}$$

$$\frac{dy}{dx} = -\sqrt{P/EI}\, A \sin\left(x\sqrt{P/EI}\right) + \sqrt{P/EI}\, B \cos\left(x\sqrt{P/EI}\right) \longrightarrow (2)$$

$\frac{dy}{dx} = 0$; $A = -a$; $x = 0$ sub in eqn(2)

$$0 = \sqrt{P/EI}\; a\, \cancel{\sin\left(0\sqrt{P/EI}\right)} + \sqrt{P/EI}\; B \cos\left(0\sqrt{P/EI}\right)$$

$$0 = \sqrt{P/EI}\; B$$

$B = 0$ $\qquad \sqrt{P/EI} \neq 0$

$A = -a$; $B = 0$ sub in General eqn

$$y = -a\cos\left(x\sqrt{P/EI}\right) + 0\,\cancel{\sin\left(x\sqrt{P/EI}\right)} + a$$

$$y = -a\cos\left(x\sqrt{P/EI}\right) + a \longrightarrow (3)$$

At a point 'R' $x = l$; $y = a$ → sub in (3)rd eqn

$$a = -a\cos\left(l\sqrt{P/EI}\right) + a$$

$$a + a\cos\left(l\sqrt{P/EI}\right) = a$$

$$a\cos\left(l\sqrt{P/EI}\right) = a - a$$

$$a\cos\left(l\sqrt{P/EI}\right) = 0$$

$a \neq 0$ apply the condition in above eqn

$$\cos\left(l\sqrt{P/EI}\right) = 0$$

$l\sqrt{P/EI} = \cos^{-1}(0)$

$\left(l\sqrt{P/EI}\right) = \pi/2$

$\sqrt{P/EI} = \pi/2l$

Squaring on both side

$$P/EI = \frac{\pi^2}{4l^2}$$

$$P = \frac{\pi^2 E.I}{4l^2}$$