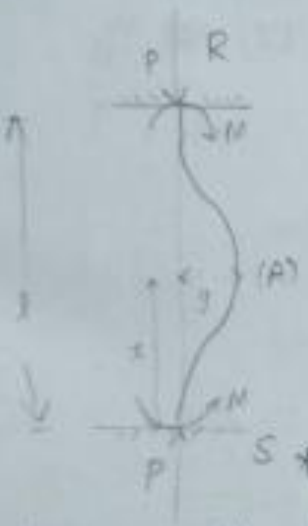
column with both ends are fixed -



consider a column R.S of length l and uniform cross sectional area a carrying a vertical load P at both ends R&S the column is fixed at both of its ends R&S due to application of load P the column will deflect as shown in figure.

Taking moment at point A

$M = M - Py \rightarrow (1)$ (-ve indicate concavity)

$M = EI \frac{d^2y}{dx^2} \rightarrow (2)$

equating (1) and (2)

$EI \frac{d^2y}{dx^2} = M - Py$

dividing the equation by EI

$$\frac{d^2y}{dx^2} = M/_{EI} - P/_{EI}\, y$$

$$\frac{d^2y}{dx^2} + P/_{EI}\, y = M/_{EI} \longrightarrow (1)$$

$\times$ with (P) on RHS

$$\frac{d^2y}{dx^2} + P/_{EI}\, y = M/_{EI} \times P/_P$$

$$\frac{d^2y}{dx^2} + P/_{EI} = P/_{EI} \times M/_P$$

The general equation

$$y = A \cos(x \sqrt{P/_{EI}}) + B \sin(x \sqrt{P/_{EI}}) + M/_P$$

At a point "s" $x = 0;\ y = 0;\ \frac{dy}{dx} = 0.$

$$0 = A \left(\cos\, (0 \sqrt{P/_{EI}}\right) + B \sin \cancel{\left(0 \sqrt{P/_{EI}}\right)}^{0} + M/_P$$

$$0 = A \cos(0) + M/_P$$

$$A = -M/_P$$

differentiate the general eqn w.r.to x;

$$\frac{dy}{dx} = -A \sin(x \sqrt{P/_{EI}}) \sqrt{\tfrac{P}{EI}} + B \cos\, (x \sqrt{P/_{EI}}) \sqrt{P/_{EI}}$$

$$\frac{dy}{dx} = -\sqrt{P/_{EI}}\, A \sin(x \sqrt{P/_{EI}}) + \sqrt{P/_{EI}}\ B \cos(x \sqrt{P/_{EI}})$$

$$0 = +\sqrt{P/_{EI}}\ \cancel{M/_P\, (\sin(0) \sqrt{P/_{EI}})} + \sqrt{\tfrac{P}{EI}}\, B \cos((0) \sqrt{P/_{EI}})$$

$$0 = \sqrt{P/_{EI}}\ B$$

$B\sqrt{P/_{EI}} = 0$ $B = 0$ $\sqrt{P/_{EI}} \neq 0$

apply this condition in

$y = -M/P \cos\left(x\sqrt{P/EI}\right) + \cancel{0 \times \sin}\left(x\sqrt{P/EI}\right) + M/P$

$y = -M/P \cos\left(x\sqrt{P/EI}\right) + M/P \rightarrow (4)$

considering the point R hence the value of

x & y is $x = l;\ y = 0$

sub in eqn (4)

$0 = -M/P \cos\left(l\sqrt{P/EI}\right) + M/P$

$M/P \cos\left(l\sqrt{P/EI}\right) = M/P$

$\cos l\left(\sqrt{P/EI}\right) = 1$

$l\sqrt{P/EI} \quad \cos^{-1}(1)$

$l\sqrt{P/EI} = 2\pi$

$\sqrt{P/EI} = 2\pi/l$

squaring on both side to cancell the root

$$P/EI = \frac{4\pi^2}{l^2}$$

$$P = \frac{4\pi^2 EI}{l^2}$$