SNS COLLEGE OF TECHNOLOGY

Coimbatore-23
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT232 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT IV – STABILITY ANALYSIS

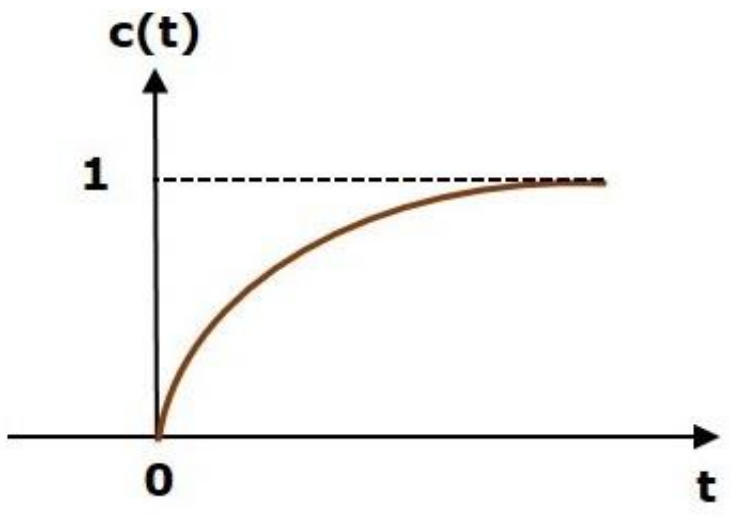
TOPIC 1,2 STABILITY & ROUTH HURWITZ CRITERION

OUTLINE

- **REVIEW ABOUT PREVIOUS CLASS**
- INTRODUCTION
- TYPES OF STABILITY
- ABSOLUTELY ,MARGINALLY & CONDITIONALLY :**STABLE SYSTEM**
- **ACTIVITY**
- **ROUTH-HURWITZ STABILITY CRITERION** INTRODUCTION
- NECESSARY CONDITION FOR ROUTH-HURWITZ STABILITY
- SUFFICIENT CONDITION FOR ROUTH-HURWITZ STABILITY
- ROUTH TABLE
- SPECIAL CASES OF ROUTH ARRAY
- HOW TO FIND STABILITY-2 CASES
- **SUMMARY**

INTRODUCTION

- A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable.
- A stable system produces a bounded output for a given bounded input.



TYPES OF STABILITY

- Stability
  - Absolutely stable system
  - Conditionally stable system
  - Marginally stable system

ABSOLUTELY STABLE SYSTEM

- If the system is stable for all the range of system component values, then it is known as the absolutely stable system.
- The **open loop** control system is absolutely **stable** if all the **poles** of the open loop transfer function present in **left half of ‘s’ plane.**
- Similarly, the **closed loop** control system is absolutely **stable** if all the poles of the closed loop transfer function present in **the left half of the ‘s’ plane**.

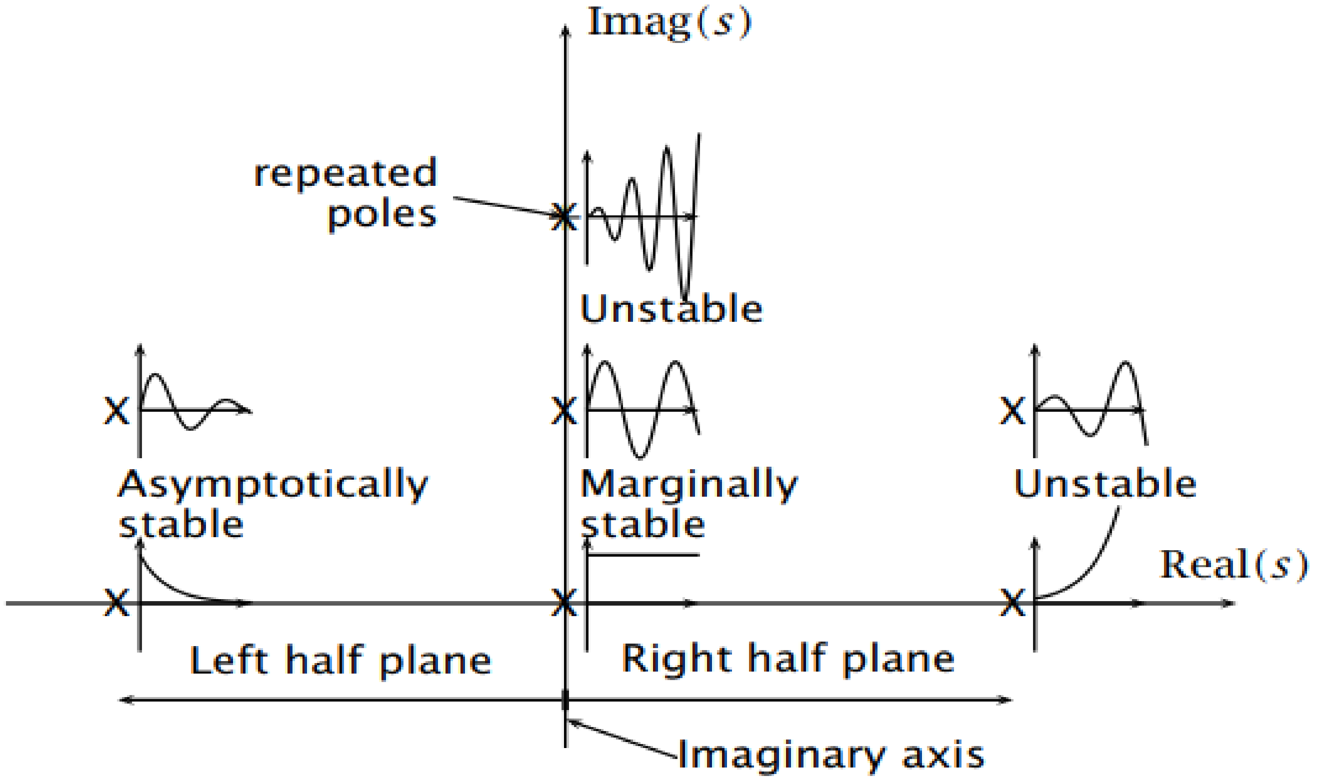
MARGINALLY STABLE SYSTEM

- If the system is stable by producing an output signal with constant **amplitude** and constant **frequency** of oscillations for bounded input, then it is known as marginally stable system.
- The **open loop** control system is marginally stable if any **two poles** of the open loop transfer function is present on the **imaginary axis.**
- Similarly, the **closed loop** control system is marginally stable if any **two poles** of the closed loop transfer function is present on the **imaginary axis.**

CONDITIONALLY STABLE SYSTEM

- If the system is stable for a certain range of system component values, then it is known as conditionally stable system.

STABLE SYSTEM



ACTIVITY

1.What 5-letter word becomes shorter when you add two letters to it?

2.What number comes next in the following sequence? 2 4 8 10 20 _

3.When can you add two to eleven and get one as the correct answer?

4.If you wrote all of the numbers from 300 to 400 on a piece of paper, how many times would you have written the number 3?

ACTIVITY-answers

1.Short

2. 22 (the sequence alternates +2, x2)

3.When you add two hours to eleven o’clock, you get one o’clock.

4.120 (100 threes in the hundreds place + 10 threes in the tens place + 10 threes in the ones place)

ROUTH-HURWITZ STABILITY CRITERION INTRODUCTION

- It is having one necessary condition and one sufficient condition for stability.
- If any control system doesn't satisfy the necessary condition, then we can say that the control system is unstable.
- But, if the control system satisfies the necessary condition, then it may or may not be stable.
- So, the sufficient condition is helpful for knowing whether the control system is stable or not.

NECESSARY CONDITION FOR ROUTH-HURWITZ STABILITY

- The necessary condition is that the **coefficients** of the characteristic polynomial should be **positive.**
- This implies that all the **roots of the characteristic equation should have negative real parts.**

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} s^1 + a_n s^0 = 0$$

- Note that, there should not be any term missing in the nth order characteristic equation.
- This means that the nth order characteristic equation should not have any coefficient that is of zero value.

SUFFICIENT CONDITION FOR ROUTH-HURWITZ STABILITY

- The sufficient condition is that all the elements of the first column of the Routh array should have the same sign.
- This means that all the elements of the first column of the Routh array should be either positive or negative.
- If all the roots of the characteristic equation exist to the left half of the ‘s’ plane, then the control system is stable.
- If at least one root of the characteristic equation exists to the right half of the ‘s’ plane, then the control system is unstable

ROUTH TABLE

s^n	a_0	a_2	a_4	a_6	$\ldots$	$\ldots$
s^{n-1}	a_1	a_3	a_5	a_7	$\ldots$	$\ldots$
s^{n-2}	$b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1}$	$b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}$	$b_3 = \frac{a_1 a_6 - a_7 a_0}{a_1}$	$\ldots$	$\ldots$	$\ldots$
s^{n-3}	$c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$	$c_2 = \frac{b_1 a_5 5 - b_3 a_1}{b_1}$	$\vdots$			
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
s^1	$\vdots$	$\vdots$				
s^0	a_n					

SPECIAL CASES OF ROUTH ARRAY

- **The first element of any row of the Routh array is zero.**
  - If any row of the Routh array contains only the first element as zero and at least one of the remaining elements have non-zero value, then replace the first element with a small positive integer, ϵ.
  - And then continue the process of completing the Routh table. Now, find the number of sign changes in the first column of the Routh table by substituting ϵ tends to zero.

SPECIAL CASES OF ROUTH ARRAY

- **All the elements of any row of the Routh array are zero.**
  - Write the auxiliary equation, A(s) of the row, which is just above the row of zeros.
  - Differentiate the auxiliary equation, A(s) with respect to s.
  - Fill the row of zeros with these coefficients.

HOW TO FIND STABILITY

$$s^4+3s^3+3s^2+2s+1=0$$

Step 1 – Verify the necessary condition for the Routh-Hurwitz stability.

All the coefficients of the characteristic polynomial, $s^4+3s^3+3s^2+2s+1$ are positive. So, the control system satisfies the necessary condition.

Step 2 – Form the Routh array for the given characteristic polynomial.

s^4	1	3	1
s^3	3	2	
s^2	$\frac{(3\times3)-(2\times1)}{3}=\frac{7}{3}$	$\frac{(3\times1)-(0\times1)}{3}=\frac{3}{3}$ $=1$	
s^1	$\frac{\left(\frac{7}{3}\times2\right)-(1\times3)}{\frac{7}{3}}$ $=\frac{5}{7}$		
s^0	1		

HOW TO FIND STABILITY...

- Step 3 – Verify the sufficient condition for the Routh-Hurwitz stability.
- All the elements of the first column of the Routh array are positive. There is no sign change in the first column of the Routh array. So, the control system is stable.

HOW TO FIND STABILITY...

$$s^4 + 2s^3 + s^2 + 2s + 1 = 0$$

Step 1 − Verify the necessary condition for the Routh-Hurwitz stability.

All the coefficients of the characteristic polynomial, $s^4 + 2s^3 + s^2 + 2s + 1$ are positive. So, the control system satisfied the necessary condition.

Step 2 − Form the Routh array for the given characteristic polynomial.

s^4	1	1	1
s^3	$\cancel{2}\ 1$	$\cancel{2}\ 1$	
s^2	$\frac{(1\times1)-(1\times1)}{1} = 0$	$\frac{(1\times1)-(0\times1)}{1} = 1$	
s^1			
s^0			

The row s^3 elements have 2 as the common factor. So, all these elements are divided by 2.

HOW TO FIND STABILITY...

Special case (i) – Only the first element of row s^2 is zero. So, replace it by ϵ and continue the process of completing the Routh table.

s^4	1	1	1
s^3	1	1	
s^2	ϵ	1	
s^1	$\frac{(\epsilon\times1)-(1\times1)}{\epsilon} = \frac{\epsilon-1}{\epsilon}$		
s^0	1		

Step 3 – Verify the sufficient condition for the Routh-Hurwitz stability.

HOW TO FIND STABILITY...

As ϵ tends to zero, the Routh table becomes like this.

s^4	1	1	1
s^3	1	1	
s^2	0	1	
s^1	$-\infty$		
s^0	1		

There are two sign changes in the first column of Routh table. Hence, the control system is unstable.

HOW TO FIND STABILITY...

$$s^5 + 3s^4 + s^3 + 3s^2 + s + 3 = 0$$

Step 1 − Verify the necessary condition for the Routh-Hurwitz stability.

All the coefficients of the given characteristic polynomial are positive. So, the control system satisfied the necessary condition.

Step 2 − Form the Routh array for the given characteristic polynomial.

s^5	1	1	1
s^4	$\cancel{3}$ 1	$\cancel{3}$ 1	$\cancel{3}$ 1
s^3	$\frac{(1\times1)-(1\times1)}{1} = 0$	$\frac{(1\times1)-(1\times1)}{1} = 0$	
s^2			
s^1			
s^0			

The row s^4 elements have the common factor of 3. So, all these elements are divided by 3.

HOW TO FIND STABILITY...

Special case (ii) – All the elements of row s^3 are zero. So, write the auxiliary equation, A(s) of the row s^4.

$$A(s) = s^4 + s^2 + 1$$

Differentiate the above equation with respect to s.

$$\frac{dA(s)}{ds} = 4s^3 + 2s$$

Place these coefficients in row s^3.

s^5	1	1	1
s^4	1	1	1
s^3	4 2	$\cancel{2}$ 1	
s^2	$\frac{(2\times1)-(1\times1)}{2} = 0.5$	$\frac{(2\times1)-(0\times1)}{2} = 1$	
s^1	$\frac{(0.5\times1)-(1\times2)}{0.5} = \frac{-1.5}{0.5}$ $= -3$		
s^0	1		

SUMMARY