SNS COLLEGE OF TECHNOLOGY

Coimbatore-20
An Autonomous Institution

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT I – CONTROL SYSTEM MODELING

TOPIC 5- MODELING OF ELECTRIC SYSTEMS

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS

- TYPES OF SYSTEMS

- DYNAMIC SYSTEMS

- WAYS TO STUDY A SYSTEM

- MODEL & ITS NEEDS,TYPES

- ACTIVITY

- MODELING OF ELECTRICAL SYSTEMS(R,L,C)

- V-I AND I-V RELATIONS

- EXAMPLES

- SUMMARY

TYPES OF SYSTEMS

- **Static System:** If a system does not change with time, it is called a static system.

- **Dynamic System:** If a system changes with time, it is called a dynamic system.
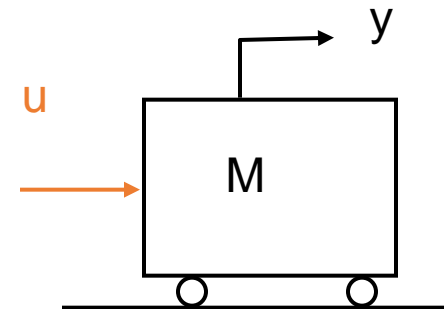
DYNAMIC SYSTEMS

- A system is said to be dynamic if its current output may depend on the past history as well as the present values of the input variables.

- Mathematically,
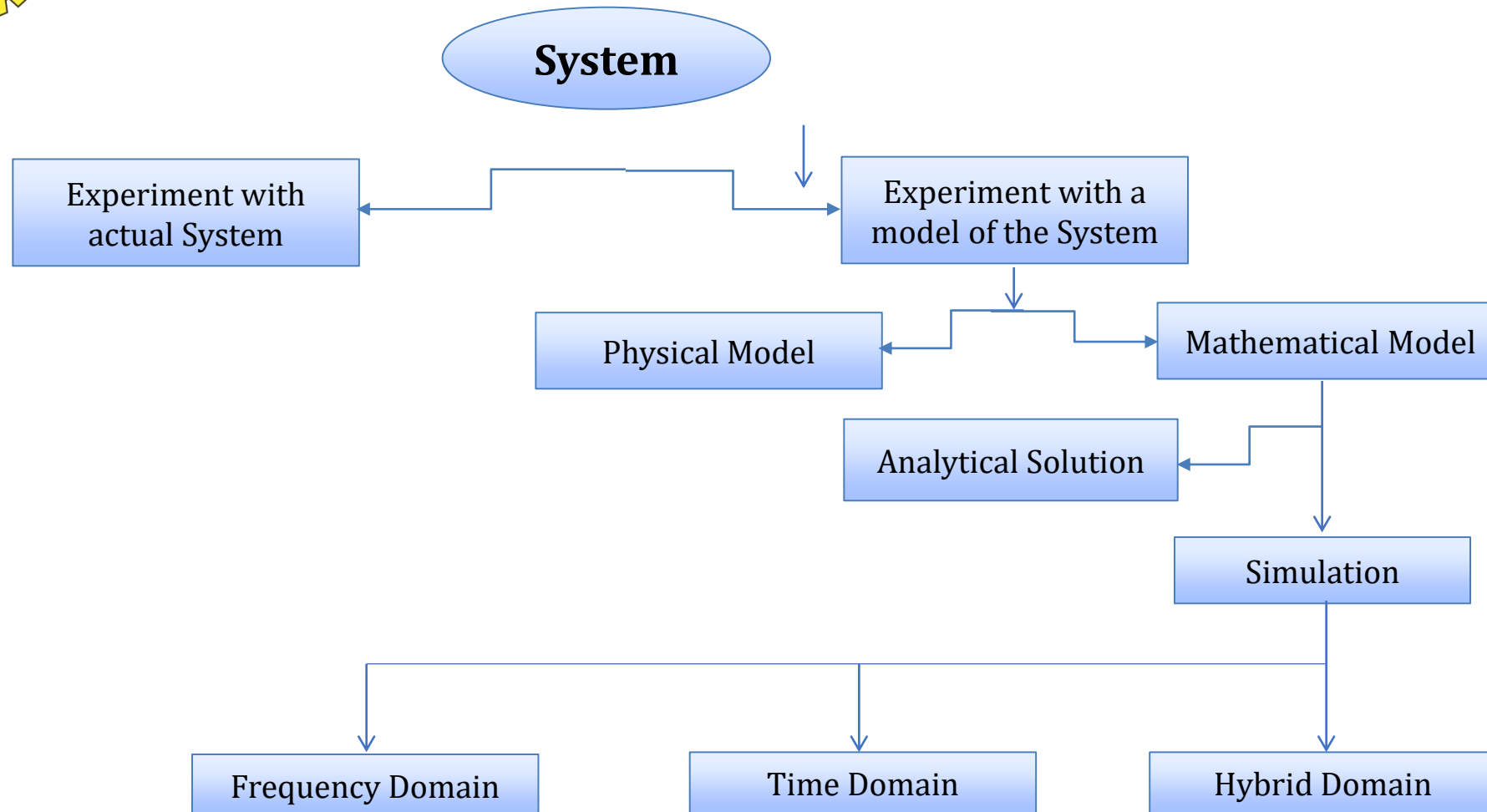
$$y(t) = \varphi[u(\tau), 0 \leq \tau \leq t]$$

$$u : \text{Input}, \ t : \text{Time}$$

Example: A moving mass

Model: Force=Mass x Acceleration

WAYS TO STUDY A SYSTEM

System

Experiment with actual System

Experiment with a model of the System

Physical Model

Mathematical Model

Analytical Solution

Simulation

Frequency Domain

Time Domain

Hybrid Domain

MODEL

- A *model* is a simplified representation or abstraction of reality.

- Reality is generally too complex to model exactly.

A set of mathematical equations (e.g., differential eqs.) that describes the input-output behavior of a system.

What is a model used for?

- Simulation

- Prediction/Forecasting

- Prognostics/Diagnostics

- Design/Performance Evaluation

- Control System Design

BLACK BOX MODEL

- When only input and output are known.
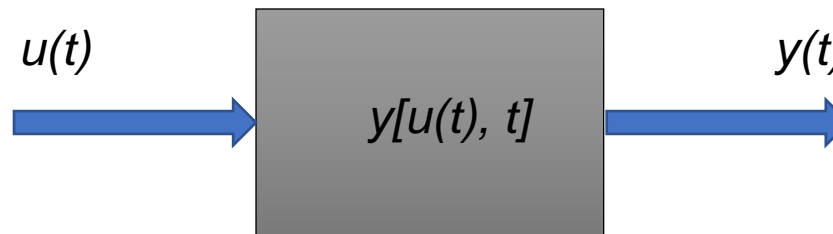- Internal dynamics are either too complex or unknown.

Input → ⬛ → Output

- Easy to Model

GREY BOX MODEL

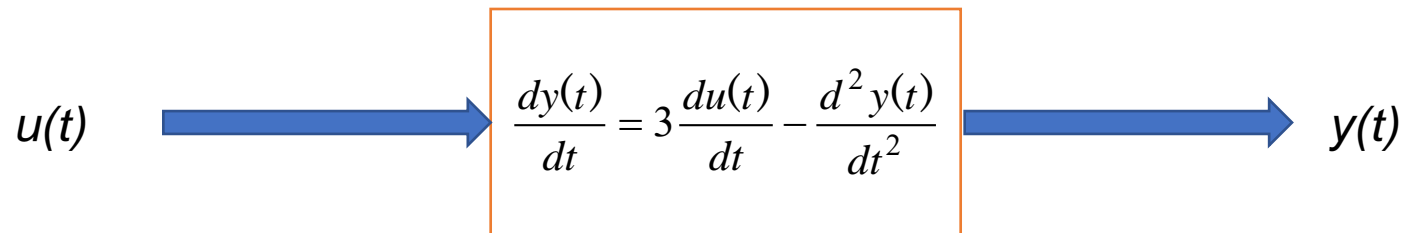- When input and output and some information about the internal dynamics of the system is known.

u(t) → [y[u(t), t]] → y(t)

- Easier than white box Modelling.

WHITE BOX MODEL

- When input and output and internal dynamics of the system is known.

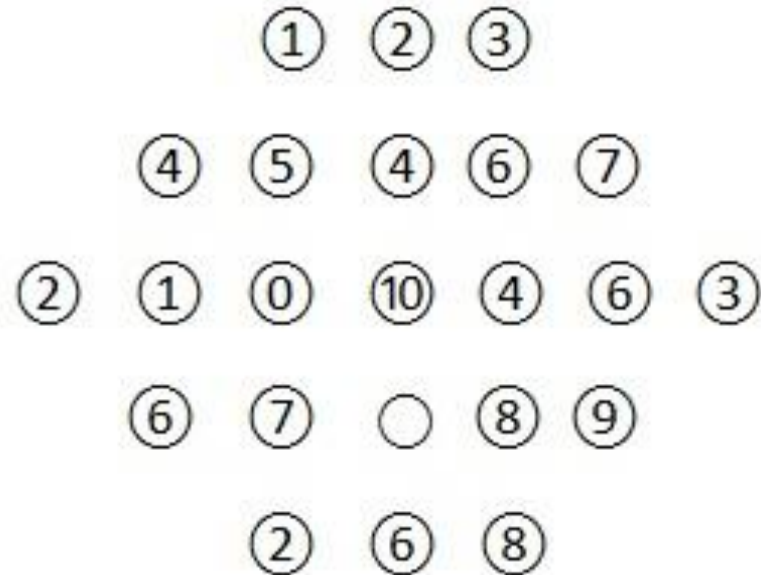$$u(t) \longrightarrow \boxed{\frac{dy(t)}{dt} = 3\frac{du(t)}{dt} - \frac{d^2 y(t)}{dt^2}} \longrightarrow y(t)$$

- One should know complete knowledge of the system to derive a white box model.

ACTIVITY

Fill the empty circle

① ② ③

④ ⑤ ④ ⑥ ⑦

② ① ⓪ ⑩ ④ ⑥ ③

⑥ ⑦ ○ ⑧ ⑨

② ⑥ ⑧

BASIC ELEMENTS OF ELECTRICAL SYSTEMS

Symbol ➤

- The time domain expression relating voltage and current for the resistor is given by Ohm's law

$$v_R(t) = i_R(t)R$$

- The Laplace transform of the above equation is

$$V_R(s) = I_R(s)R$$

Capacitor

- The time domain expression relating voltage and current for the Capacitor is given as:

$$v_c(t) = \frac{1}{C}\int i_c(t)dt$$

- The Laplace transform of the above equation (assuming there is no charge stored in the capacitor) is

$$V_c(s) = \frac{1}{Cs}I_c(s)$$

BASIC ELEMENTS OF ELECTRICAL SYSTEMS

Inductor

- The time domain expression relating voltage and current for the inductor is given as:

$$v_L(t) = L\frac{di_L(t)}{dt}$$

- The Laplace transform of the above equation (assuming there is no energy stored in inductor) is
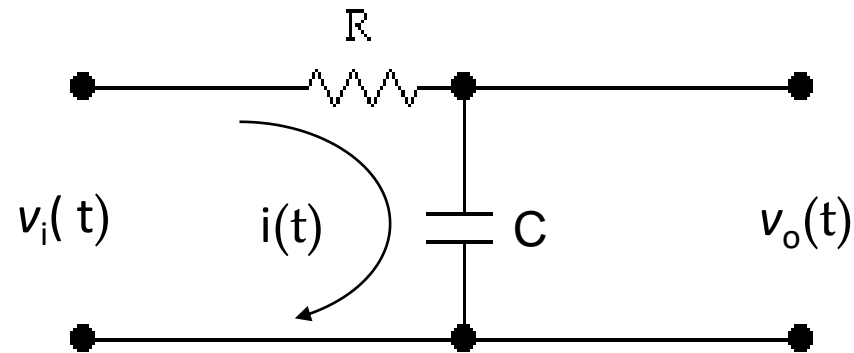
$$V_L(s) = LsI_L(s)$$

V-I AND I-V RELATIONS

Component	Symbol	V-I Relation	I-V Relation
Resistor		$v_R(t) = i_R(t)R$	$i_R(t) = \dfrac{v_R(t)}{R}$
Capacitor		$v_c(t) = \dfrac{1}{C}\int i_c(t)dt$	$i_c(t) = C\dfrac{dv_c(t)}{dt}$
Inductor		$v_L(t) = L\dfrac{di_L(t)}{dt}$	$i_L(t) = \dfrac{1}{L}\int v_L(t)dt$

EXAMPLE 1

- The two-port network shown in the following figure has $v_i(t)$ as the input voltage and $v_o(t)$ as the output voltage. Find the transfer function $V_o(s)/V_i(s)$ of the network.

$$v_i(t) = i(t)R + \frac{1}{C}\int i(t)dt \qquad\qquad v_o(t) = \frac{1}{C}\int i(t)dt$$

EXAMPLE 1

$$v_i(t) = i(t)R + \frac{1}{C}\int i(t)dt \qquad v_o(t) = \frac{1}{C}\int i(t)dt$$

- Taking Laplace transform of both equations, considering initial conditions to zero.

$$V_i(s) = I(s)R + \frac{1}{Cs}I(s) \qquad V_o(s) = \frac{1}{Cs}I(s)$$

- Re-arrange both equations as:

$$V_i(s) = I(s)(R + \frac{1}{Cs}) \qquad CsV_o(s) = I(s)$$

EXAMPLE 1

$$V_i(s) = I(s)(R + \frac{1}{Cs}) \qquad\qquad CsV_o(s) = I(s)$$

- Substitute I(s) in equation on left

$$V_i(s) = CsV_o(s)(R + \frac{1}{Cs})$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{Cs(R + \frac{1}{Cs})}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + RCs}$$

EXAMPLE 1

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + RCs}$$

- The system has one pole at

$$1 + RCs = 0 \qquad \Rightarrow s = -\frac{1}{RC}$$

EXAMPLE 2

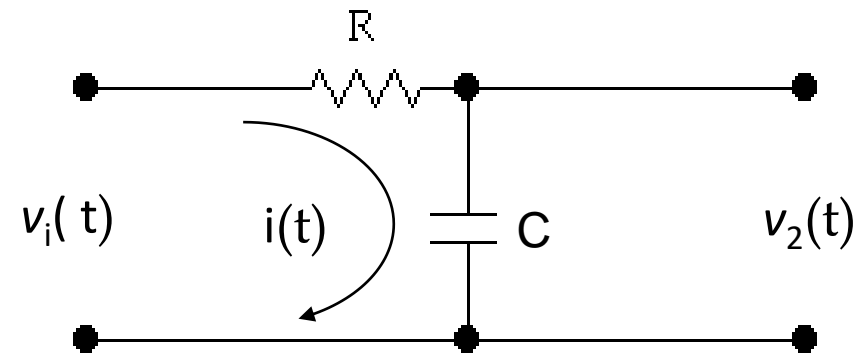- Design an Electrical system that would place a pole at -3 if added to the other system.

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + RCs}$$

- System has one pole at

$$s = -\frac{1}{RC}$$

- Therefore,

$$-\frac{1}{RC} = -3 \qquad if \qquad R = 1 \; M\Omega \quad and \quad C = 333 \; pF$$

SUMMARY