



Weighted residual formulation

Solve the differential Equation for a physical problem. expressed as $\frac{d^2y}{dx^2} + 100 = 0$, $0 \leq x \leq 10$. with boundary conditions as $y(0) = 0$, $y(10) = 0$, $x(0) = 0$. using

(i) Point collection method (ii) sub-domain collection method.
(iii) Least square method (iv) Galerkin's method.

Sol Given data

$$\frac{d^2y}{dx^2} + 100 = 0 \quad \text{--- (1)} \quad 0 \leq x \leq 10$$

x is range of 0 to 10
Boundary condition $y(0) = 0$
 $y(10) = 0$

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Polynomial function, $y = a_1 + a_2x + a_3x^2$ --- (2)

boundary condition $x=0, y=0$

$$y(0) = a_1 + a_2(0) + a_3(0)^2$$
$$0 = a_1$$
$$\boxed{a_1 = 0}$$

let $x=10, y=0$

$$y(10) = a_1 + a_2x + a_3x^2$$
$$0 = a_1 + a_2(10) + a_3(10)^2$$
$$0 = 10a_2 + 100a_3$$
$$100 = -100a_3$$

$$\therefore a_1 = 0 \quad a_2 = -10a_3$$

a_1, a_2 values are substituted in Eq (2)

$$y = a_1 + a_2 x + a_3 x^2$$

$$= 0 + (-10a_3)x + a_3 x^2$$

$$= -10a_3 x + a_3 x^2$$

$$= a_3 x [x - 10]$$

$$= ax(x - 10) \quad (\because a_3 = a)$$

hence trial function $y = ax(x - 10)$

now, differentiate the above Equation w.r.t x

$$y = ax^2 - 10ax$$

$$\frac{dy}{dx} = a(2x) - 10a(1)$$

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$$\frac{dy}{dx} = 2ax - 10a$$

again differentiate the above Equation w.r.t x .

$$\frac{d^2y}{dx^2} = 2a(1) - 10a(0)$$

$$\frac{d^2y}{dx^2} = 2a$$

Substitute $\frac{d^v y}{dx^v}$ value in Eq (1) $\frac{d^v y}{dx^v} + 100$

$$R = \underline{2a + 100}$$