SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF AEROSPACE ENGINEERING

19ASB304 – COMPUTATIONAL FLUID DYNAMICS FOR AEROSPACE APPLICATIONS
III YEAR VI SEM
UNIT-I FUNDAMENTAL CONCEPTS
TOPIC: Discretization of partial differential equations

NAME: Mr.N.Venkatesh., M.Tech
Assistant Professor
Aerospace Engineering
SNS College of Technology

FORMATION OF PARTIAL DIFFERENTIAL EQUATIONS

- It is defined as an equation involving two or more independent variables like x,y……., a dependent variable like u and its partial derivatives.
- Partial Differential Equation can be formed either by elimination of arbitrary constants or by the elimination of arbitrary functions from a relation involving three or more variables .

GENERAL FORM

- The general form of a first order partial differential equation is

$$F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}) = F(x, y, z, p, q) = 0......(1)$$

where x, y are two independent variables, z is the dependent variable and $p = z_x$ and $q = z_y$

DIFFERENT INTEGRALS OF PARTIAL DIFFERNETIAL EQUATION

1) COMPLETE INTEGRAL SOLUTION
2) PARTICULAR SOLUTION
3) SINGULAR SOLUTION
4) GENERAL SOLUTION

COMPLETE SOLUTION

- Let $F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}) = F(x, y, z, p, q) = 0......(1)$

  be the Partial Differential Equation.

- The complete integral of equation (1) is given by

$$\phi(x, y, z, a, b) = 0.........(2)$$

- where a and b are two arbitrary constants

PARTICULAR SOLUTION

- A solution obtained by giving the particular values to the arbitrary constants in a complete integral is called particular solution.

SINGULAR SOLUTION

- It is the relation between those specific variables which involves no arbitrary constant and is not obtainable as a particular integral from the complete integral.
- So, equation is

$$\phi(x, y, z, a, b) = 0$$

$$\frac{\partial \phi}{\partial a} = 0, \frac{\partial \phi}{\partial b} = 0$$

GENERAL SOLUTION

- A relation between the variables involving two independent functions of the given variables together with an arbitrary function of these variables is a general solution.

- In this given equation

$$\phi(x, y, z, a, b) = 0 \ldots\ldots\ldots (2)$$

assume an arbitrary relation of form $b = f(a)$

- So, our earlier equation becomes

$$\phi(x, y, z, a, f(a)) = 0 \ldots\ldots\ldots (3)$$

- Now, differentiating (2) with respect to a and thus we get,

$$\frac{\partial \phi}{\partial a} + \frac{\partial \phi}{\partial b} f'(a) = 0 \ldots\ldots\ldots (4)$$

- If the eliminator of (3) and (4) exists, then it is known as general solution.

STANDARD TYPES OF FIRST ORDER EQUATIONS

- **TYPE-1**

The Partial Differential equation of the form

$$f(p,q)=0$$

has solution

$$z=ax+by+c \text{ and } f(a,b)=0$$

- TYPE-2

The partial differentiation equation of the form

$$z = ax + by + f(a,b)$$

is called *Clairaut's* form of partial differential equations.

- TYPE-3
- If the partial differential equations is given by

$$f(z, p, q) = 0$$

- Then assume that

$$z = \phi(x + ay)$$
$$u = x + ay$$
$$z = \phi(u)$$

$$p = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} = \frac{\partial z}{\partial u}.1 = \frac{dz}{du}$$

$$q = \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} = \frac{\partial z}{\partial u}.a = a\frac{dz}{du}$$

- TYPE-4
- The partial differential equation of the given form can be solved by assuming

$$f(x,p) = g(y,q) = a$$

$$f(x,p) = a \Rightarrow p = \phi(x,a)$$

$$g(y,q) = a \Rightarrow q = \Psi(y,a)$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$dz = \phi(x,a)dx + \Psi(y,a)dy$$

EXAMPLES

- 1.Solve the *pde* $p^2 - q = 1$ and find the complete and singular solutions
- Solution

  Complete solution is given by

- $$z = ax + by + c$$ with

$$a^2 - b = 1$$

$$\Rightarrow b = a^2 - 1$$

$$z = ax + (a^2 - 1)y + c$$

d.w.r.to. a and c then

$$\frac{\partial z}{\partial a} = x + 2ay$$

$$\frac{\partial z}{\partial c} = 1 = 0$$ Which is not possible

Hence there is no singular solution

- 2.Solve pde $pq = xy$

(or) $(\frac{\partial z}{\partial x})(\frac{\partial z}{\partial y}) = xy$

Solution $\frac{p}{x} = \frac{y}{q}$

Assume that

$$\frac{p}{x} = \frac{y}{q} = a$$

$$\therefore p = ax, q = \frac{y}{a}$$

$$dz = pdx + qdy = axdx + \frac{y}{a}dy$$

- Integrating on both sides

$$z = a\frac{x^2}{2} + \frac{y^2}{2a} + b$$