UNIT-I VECTOR CALCULUS
UNIT-I VECTOR CALCULUS
DERIVATIVES: Gradient of a scalar field, Directional Derivative unit- II
vector e calculus
Gradient :
Let $\phi(x, y, z)$ be a scalar point function and is continuously differentiable. Then the vector
$\nabla \phi=\vec{i} \frac{\partial \phi}{\partial x}+\vec{\delta} \frac{\partial \phi}{\partial y}+\vec{k} \frac{\partial \phi}{\partial z}$ is called the gradient of the scalar for $\phi$.

$$
\text { ie, } \operatorname{grad} \phi=\nabla \phi
$$

Problems
I. Find $\nabla \phi$ whole $\quad \phi=x^{2}+y^{2}+z^{2}$ Sol.

$$
\begin{aligned}
& \text { Grad } \phi \text { (or) } \nabla \phi=\vec{i} \frac{\partial \phi}{\partial x}+\vec{j} \frac{\partial \phi}{\partial y}+\vec{k} \frac{\partial \phi}{\partial z} \\
& \quad=\overrightarrow{1} \frac{\partial}{\partial x}\left(x^{2}+y^{2}+z^{2}\right)+\vec{j} \frac{\partial}{\partial y}\left(x^{2}+y^{2}+z^{2}\right)+\vec{k} \frac{\partial}{\partial z}\left(x^{2}+y^{2}+z^{2}\right.
\end{aligned}
$$

$$
=T(2 x)+\vec{j}(2 y)+\overrightarrow{r^{\prime}}(2 x)
$$

$$
\nabla \phi=2 x \vec{\imath}+2 y \vec{\jmath}+2 z k
$$

2]. Find $\nabla \phi$ whore $\bar{\phi}=3 x^{2} y-y^{3} z^{2}$ at $(1,1,1)$
Sols.

$$
\begin{aligned}
\text { Son. } \\
\begin{aligned}
\nabla \phi & =\vec{l} \frac{\partial \phi}{\partial x}+\vec{j} \frac{\partial \phi}{\partial y}+\vec{k} \frac{\partial \phi}{\partial z} \\
& =\vec{T} \frac{\partial}{\partial x}\left(3 x^{2} y-y^{3} z^{2}\right)+\vec{j} \frac{\partial}{\partial y}\left(3 x^{2} y-y^{3} z^{2}\right)+\vec{k} \frac{\partial}{\partial z}\left(3 x^{2} y-y^{3} z^{2}\right) \\
& =\rightarrow \overrightarrow{1}[6 x y-0]+\vec{j}^{4}\left[3 x^{2}-3 y^{2} z^{2}\right]+\vec{k}\left[0-2 y^{3} z\right] \\
\nabla \phi & =6 x y \overrightarrow{1}+\left(3 x^{2}-3 y^{2} z^{2}\right) \vec{j}+2 y^{2} z \vec{k} \\
\nabla \phi(1,1,1) & =6(1)(1) \overrightarrow{1}+(3-3) \vec{j}-2(1)(1) \vec{k} \\
& =6 \overrightarrow{1}+0 \vec{\delta}-2 \vec{k}
\end{aligned}
\end{aligned}
$$

CS Scanned with CamScanne $=6 \vec{T}-2 \vec{K}$
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UNIT-I VECTOR CALCULUS
DERIVATIVES: Gradient of a scalar field, Directional Derivative 3]. Find the maximum directional derivative of $\phi=x y z^{2}$ at $(1,0,3$ ) Sols.

$$
\begin{aligned}
\nabla \phi & =\vec{r} \frac{\partial \phi}{\partial x}+\vec{J} \frac{\partial \phi}{\partial y}+\vec{k} \frac{\partial \phi}{\partial x} \\
& =\vec{r} \frac{\partial}{\partial x}\left(x y z^{2}\right)+\vec{J} \frac{\partial}{\partial y}\left(x y z^{2}\right)+\vec{k} \frac{\partial}{\partial z}\left(x y z^{2}\right) \\
\nabla \phi & =\vec{T}\left(y z^{2}\right)+\vec{J}\left(x z^{2}\right)+\vec{k}(x y z z) \\
\nabla \phi(1,0,3) & =\vec{r}(0)+\vec{J}(1)(9)+\vec{k}(0) \\
& =9 \vec{\delta} \quad \text { maximum } D D=\sqrt{91}=8
\end{aligned}
$$

47. Find $\nabla \phi$ where $\phi=x y z$ at ( $1,2,3$ )

Sols.

$$
\begin{aligned}
\nabla \phi & =\vec{r} \frac{\partial \phi}{\partial x}+\vec{j} \frac{\partial \phi}{\partial y}+\vec{k} \frac{\partial \phi}{\partial z} \\
& =\vec{r} \frac{\partial(x y z)}{\partial x}+\vec{d} \frac{\partial(x y z)}{\partial y} . \vec{k} \frac{\partial(x y z)}{\partial z}
\end{aligned}
$$

$\nabla \phi=\vec{T}(y z)+\vec{\jmath}(x z)+\vec{k}(x y)$

$$
\begin{aligned}
\nabla()_{(1,2,3)} & =\vec{r}(2)(3)+\vec{\jmath}(1)(3)+\vec{k}(1)(2) \\
& =6 \vec{r}+3 \vec{\jmath}+2 \vec{k}
\end{aligned}
$$

5]. If $\nabla \phi=y z \vec{T}+z x \vec{\jmath}+x y \vec{r}$, find $\phi$ sols.

$$
\begin{aligned}
& \nabla \phi=\vec{r} \frac{\partial \phi}{\partial x}+\vec{\jmath} \frac{\partial \phi}{\partial y}+\vec{k} \frac{\partial \phi}{\partial z} \\
& \nabla \phi=\vec{r}(y z)+\vec{J}(z x)+\vec{k}(x y)
\end{aligned}
$$

Equating $\omega, r$. to $\vec{r}, \vec{j}, \vec{k}$

$$
\begin{array}{c|l|l}
\frac{\partial \phi}{\partial x}=y z & \frac{\partial \phi}{\partial y}=z x & \frac{\partial \phi}{\partial z}=x y \\
\text { Integrate war. to } x & \text { w.r. to } y & \text { w.r. to } z \\
\phi=x y z+f(y, z) & \phi=x y z+f(x, z) & \phi=x y z+f(x, y) \\
\text { In general, }
\end{array}
$$

$$
\begin{aligned}
& \text { In general, } \\
& \text { s) scanned with } \phi=x y z+c
\end{aligned}
$$

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UNIT-I VECTOR CALCULUS
DERIVATIVES: Gradient of a scalar field, Directional Derivative
6]. If $\vec{\gamma}=x \vec{\imath}+y \vec{\jmath}+z \vec{k}$, such that $|\vec{r}|=r$, prove that

$$
\begin{aligned}
& \text { i). } \nabla r=\frac{\vec{\gamma}}{\gamma}=\hat{\gamma} \\
& \text { ii) } \nabla\left(\frac{1}{r}\right)=\frac{-\vec{\gamma}}{r^{3}}=\frac{-\hat{\gamma}}{r^{2}}
\end{aligned}
$$

$$
\text { iii). } \nabla r^{n}=n r^{n-2} \vec{\gamma}
$$

$$
\text { iv). } \nabla f(\gamma)=f^{\prime}(\gamma) \nabla \gamma
$$

Soln.
Given $\vec{r}=x \vec{\imath}+y \vec{\jmath}+z \vec{k}$

$$
\begin{aligned}
& \Rightarrow r=|\vec{\gamma}|=\sqrt{x^{2}+y^{2}+z^{2}} \\
& \quad \Rightarrow r^{2}=x^{2}+y^{2}+z^{2} \rightarrow(1)
\end{aligned}
$$

Diff. (i) w.r. to $x, y, z$,

$$
\begin{array}{r|r|r}
2 r \frac{\partial r}{\partial x}=2 x & \text { Qr } \frac{\partial r}{\partial y}=2 y & 2 r \frac{\partial r}{\partial z}=2 z \\
\frac{\partial r}{\partial x}=\frac{x}{r} & \frac{\partial u}{\partial y}=\frac{y}{r} & \frac{\partial r}{\partial z}=\frac{z}{r}
\end{array}
$$

i). $\nabla r=\vec{r} \frac{\partial r}{\partial x}+\vec{j} \frac{\partial r}{\partial y}+\vec{k} \frac{\partial r}{\partial z}$

$$
\begin{aligned}
& =\vec{r}\left(\frac{x}{r}\right)+\vec{j}\left(\frac{y}{r}\right)+\vec{k}\left(\frac{z}{\gamma}\right) \\
& =\frac{x \vec{\imath}+y \vec{\jmath}+\vec{k}}{\gamma}
\end{aligned}
$$

$$
\nabla \gamma=\frac{\vec{\gamma}}{\gamma}
$$

ii). $\nabla\left(\frac{1}{\gamma}\right)=\vec{T} \frac{\partial}{\partial x}\left(\frac{1}{\gamma}\right)+\vec{\jmath} \frac{\partial}{\partial y}\left(\frac{1}{\gamma}\right)+\vec{k} \frac{\partial}{\partial z}\left(\frac{1}{\gamma}\right)$

$$
\overrightarrow{1}\left[-\frac{1}{r^{2}} \frac{\partial r}{\partial x}\right]+\vec{j}\left[-\frac{1}{r^{2}} \frac{\partial r}{\partial y}\right]+\vec{k}\left[\frac{-1}{r^{2}} \frac{\partial r}{\partial z}\right]
$$

$$
=\vec{F}\left[\frac{-1}{\gamma^{2}} \times \frac{x}{r}\right]+\vec{j}\left[\frac{-1}{r^{2}} \times \frac{y}{r}\right]+\vec{k}\left[\frac{-1}{r^{2}} \times \frac{z}{r}\right]
$$

$$
=-\frac{1}{\gamma^{3}}[x \vec{\imath}+y \vec{\jmath}+z \vec{k}]
$$

CS $\nabla\left(\frac{1}{\gamma}\right)$ ant ir $\vec{\gamma}$

UNIT-I VECTOR CALCULUS
DERIVATIVES: Gradient of a scalar field, Directional Derivative
iii).

$$
\begin{aligned}
& \nabla r^{n}=\vec{r} \frac{\partial\left(r^{n}\right)}{\partial x}+\vec{j} \frac{\partial\left(r^{n}\right)}{\partial y}+\vec{k} \frac{\partial}{\partial z}\left(\dot{r}^{n}\right) \\
& =\vec{r} n r^{n-1} \frac{\partial r}{\partial x}+\vec{\jmath} n r^{n-1} \frac{\partial r}{\partial y}+\vec{k}^{4} n r^{n-1} \frac{\partial r}{\partial z} \\
& =n r^{n-1}\left[T \frac{\partial r}{\partial x}+\vec{j} \frac{\partial r}{\partial y}+\vec{k}^{\prime} \frac{\partial r}{\partial z}\right] \\
& =n \gamma^{n-1}\left[\vec{r}\left(\frac{x}{\gamma}\right)+\vec{\gamma}\left(\frac{y}{r}\right)+\vec{H}^{y}\left(\frac{z}{r}\right)\right] \\
& =\frac{n \gamma^{n-1}}{r}[x \vec{\imath}+y \vec{\jmath}+x \vec{k}] \\
& =\frac{n \gamma^{n-1}}{\gamma} \vec{\gamma} \\
& \nabla r^{n}=n \gamma^{n-2} \vec{r} \\
& \text { iv). } \nabla f(\gamma)=\vec{T} \frac{\partial}{\partial x} f(\gamma)+\vec{j} \frac{\partial}{\partial y} f(\gamma)+\vec{k} \frac{\partial}{\partial z} f(\gamma) \\
& =\vec{T} f^{\prime}(r) \frac{\partial r}{\partial x}+\vec{\jmath} f^{\prime}(r) \frac{\partial r}{\partial y}+\vec{k} f^{\prime}(r) \frac{\partial r}{\partial z} \\
& =f^{\prime}(\gamma)\left[\vec{\imath}\left(\frac{x}{r}\right)+\vec{\gamma}\left(\frac{y}{r}\right)+\vec{k}\left(\frac{z}{r}\right)\right] \\
& =\frac{f^{\prime}(\gamma)}{\gamma}[x \vec{\imath}+y \vec{\jmath}+z \vec{k}] \\
& =f^{\prime}(\gamma) \times \frac{\vec{\gamma}}{\gamma} \\
& \nabla f(r)=f^{\prime}(\gamma) \nabla \gamma \text { (from (i)) }
\end{aligned}
$$

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DERIVATIVES: Gradient of a scalar field, Directional Derivative
Surfaces:

$$
\begin{aligned}
\text { unit normal vector } \hat{n} & =\frac{\nabla \phi}{\mid \nabla \phi 1} \\
\text { Normal derivative } & =\mid \nabla \phi 1 \\
\text { directional derivative } & =\nabla \phi \cdot \frac{\vec{a}}{|\vec{a}|}
\end{aligned}
$$

Angle between the surfaces:

$$
\cos \theta=\frac{\nabla \phi_{1} \cdot \nabla \phi_{2}}{\left|\nabla \phi_{1}\right|\left|\nabla \phi_{2}\right|}
$$

Ifntwo surfaces are ut orthogonally, then $\nabla \phi_{1} \cdot \nabla \phi_{2}=0$
U. Fard the unit normal to the scerface

$$
\text { Find the unit bolmal }(1,-1,2) \text { at }
$$

Sols.

$$
\text { Let } \phi=x^{2}+x y+z^{2}-4
$$

unit normal vector $\hat{n}=\frac{\nabla \phi}{|\nabla \phi|}$
Now,

$$
\nabla \phi=\vec{T} \frac{\partial \phi}{\partial x}+\vec{\partial} \frac{\partial \phi}{\partial y}+\varpi \frac{\partial \phi}{\partial z}
$$

$$
\begin{aligned}
=\overrightarrow{1} \frac{\partial}{\partial \phi}\left(x^{2}+x y+z^{2}-4\right) & +\vec{d} \frac{\partial}{\partial y}\left(x^{2}+x y+z^{2}-4\right) \\
& +\vec{k}\left(x^{2}+x y+z^{2}-4\right)
\end{aligned}
$$

$$
+\vec{K} \frac{\partial}{\partial z}\left(x^{2}+x y+z^{2}-4\right)
$$

$$
=\vec{r}(2 x+y)+\vec{j}(x)+\vec{k}(2 \pi)
$$

$$
r \phi_{(1,-1,2)}=\vec{T}(2(1)-1)+\vec{\jmath}(1)+\vec{k}^{\cdots}(2(2))
$$

$$
\therefore \quad \hat{n}=\frac{\vec{r}+\vec{j}+4 \vec{k}}{\sqrt{1+1+16}}=\frac{\vec{r}+\vec{j}+4 \vec{k}}{\sqrt{18}}
$$

2]. Find the directional derivative of $\phi=x y z$ at SCi, li, wi wi the direct en of $\vec{T}+\vec{j}+\vec{k}$

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sols.

$$
\begin{aligned}
\nabla \phi & =\vec{r} \frac{\partial \phi}{\partial x}+\vec{d} \frac{\partial \phi}{\partial y}+\vec{k} \frac{\partial \phi}{\partial z} \\
\nabla \phi & =\vec{T}(y z)+\vec{d}(x z)+\vec{k}(x y) \\
\nabla \phi(1,1,1) & =\vec{T}(1)(1)+\vec{\delta}(1)(1)+\vec{k}(1)(1) \\
& =\vec{r}+\vec{j}+\vec{k}
\end{aligned}
$$

Given $\vec{a}=\vec{r}+\vec{\jmath}+\vec{k}$

$$
|\vec{a}|=\sqrt{1+1+1}=\sqrt{3}
$$

$$
D D=\nabla \phi \cdot \frac{\vec{a}}{|\vec{\nabla}|}
$$

$$
=\left(\vec{T}+\vec{J}+\overrightarrow{k^{\prime}}\right) \cdot \frac{(\vec{T}+\vec{\jmath}+\vec{k})}{\sqrt{3}}
$$

$$
=\frac{1+1+1}{\sqrt{3}}=\frac{3}{\sqrt{3}}
$$

$$
D D=\sqrt{3}
$$

3]. Find the directional derivative of
$\phi=x^{2}+2 x y$ at $(1,-1,3)$ in the direction of

$$
\overrightarrow{T_{0}}+2 \vec{\jmath}+2 \vec{k}
$$

Soln.

$$
\vec{\nabla} \phi=\vec{T} \frac{\partial \phi}{\partial x}+\vec{\delta} \frac{\partial \phi}{\partial y}+\vec{k} \frac{\partial \phi}{\partial z}
$$

$$
=\vec{r}(2 x+2 y)+\vec{J}(2 x)+\vec{k}(0)
$$

$$
\nabla \phi=(2 x+2 y) \vec{\imath}+2 x \vec{J}
$$

$$
\begin{aligned}
\nabla \phi & =(2 x+2) \\
\nabla \phi(1,-1,3) & =(2(1)+2(-1)) T+2(-1) \vec{\jmath} \\
& =-2 \vec{\jmath}
\end{aligned}
$$

$$
\vec{a}=\vec{t}+2 \vec{\jmath}+2 \vec{k}
$$

$$
|\vec{a}|=\sqrt{1+A+4}
$$

$$
=\sqrt{9}=3
$$

$$
\begin{aligned}
& =\sqrt{a}=3 \\
D D & =\nabla \phi \cdot \frac{\vec{a}}{|\vec{a}|}=-2 \vec{\jmath}-\frac{\vec{i}+2 \vec{\jmath}+2 \vec{k}}{3}
\end{aligned}
$$

$$
=-\frac{4}{3}
$$

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DERIVATIVES: Gradient of a scalar field, Directional Derivative
4]. what is the greatest, rate of increase soln.

$$
\begin{aligned}
& \nabla u=\vec{r} \frac{\partial u}{\partial x}+\vec{j} \frac{\partial u}{\partial y}+\vec{k} \frac{\partial u}{\partial z} \\
&=\vec{r}(2 x)+\vec{j}\left(z^{2}\right)+\vec{k}(2 y z) \\
& \nabla u=2 x \vec{r}+\vec{r}^{2} \vec{j}+2 y z \vec{k} \\
& \nabla u(3) \vec{k} \\
&=2 \vec{T}+9 \vec{\jmath}+2 \vec{k}(3)(3) \\
&=2 \vec{k}+9 \vec{j}-b \vec{k}
\end{aligned}
$$

$$
\begin{aligned}
& \text { The } 2 \vec{T}+a \vec{\jmath}-6 \vec{k} \\
& \therefore \text { greatest rate increase in the }
\end{aligned}
$$ direction of $y$.

5] far the angle btw the barmals to fire 5]. Find the angle bTw the baemais to is $(-3,-3,3)$ solon.

## CS <br> Scanned with $\operatorname{cosss}^{-1} \theta \equiv \cos ^{-1}(1 / \sqrt{22})$

$$
\begin{aligned}
& \text { liven } x y=z^{2} \\
& \phi=x y-z^{2} \\
& \nabla \phi=\vec{T} \frac{\partial \phi}{\partial x}+\vec{d} \frac{\partial \phi}{\partial y}+\vec{k} \frac{\partial \phi}{\partial z} \\
& =\vec{t}(y)+\vec{J}(x)+\vec{k}(-2 X) \\
& =y \vec{r}+x \vec{\jmath}-2 x \vec{k} \\
& \nabla d_{1}(1,4,2)=4 \vec{l}+\vec{J}-4 \vec{r} \\
& \mid \nabla \phi_{1} 1=\sqrt{16+1+16}=\sqrt{33} \\
& \text { and } \nabla \phi_{2}(-3,-3,3)=-3 \vec{\jmath} \vec{\jmath}-6 \vec{h} \\
& \left|\nabla \phi_{2}\right|=\sqrt{9+9+36}=\sqrt{54}=3 \sqrt{6} \\
& \therefore \cos \theta=\frac{\nabla \phi_{1}-\nabla \phi_{2}}{1 \nabla \phi_{1} 11 \nabla \phi_{2} \mid} \\
& =\frac{(4 \vec{r}+\vec{j}-4 \vec{k}) \cdot(-3 \vec{r}-3 \vec{j}-6 \vec{k})}{\sqrt{33} 3 \sqrt{6}} \\
& =\frac{-12-3+24}{3 \sqrt{11 \times 3 \times 3 \times 2}}=\frac{9}{3 \times 3 \sqrt{22}}=\frac{1}{\sqrt{22}}
\end{aligned}
$$

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UNIT-I VECTOR CALCULUS
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(j]. Find the angle betuocn the surfaces

$$
x^{2}-y^{2}-z^{2}=11 \text { and } x y+y z-z x=18 \text { at }(6,4,3)
$$

Son.
Let $\phi_{1}=x^{2}-y^{2}-z^{2}-11$

$$
\begin{aligned}
\nabla \phi_{1} & =\vec{r} \frac{\partial \phi_{1}}{\partial x}+\vec{j} \frac{\partial \phi_{1}}{\partial y}+\vec{k} \frac{\partial \phi_{1}}{\partial z} \\
& =\vec{T}(2 x)+\vec{j}(-2 y)+\vec{k}(-2 z)
\end{aligned}
$$

$$
\nabla \phi_{1}(6,4,3)=12 \vec{r}-8 \vec{\jmath}-6 \vec{k} \Rightarrow 1 \nabla q_{1} 1=\sqrt{144+64+36}
$$

$$
\text { and } \quad \phi_{2}=x y+y z-z x-18
$$

$$
=\sqrt{244}
$$

$$
\nabla \phi_{2}=\vec{r} \frac{\partial \phi_{2}}{\partial x}+\vec{j} \frac{\partial \phi_{2}}{\partial y}+\vec{k} \frac{\partial \phi_{2}}{\partial z}
$$

$$
=\vec{r}(y-z)+\vec{d}(x+z)+\vec{k}(y-x)
$$

$$
\nabla \phi_{2}(6,4,3)=\vec{r}+9 \vec{\jmath}-2 \vec{k} \Rightarrow\left|\nabla \phi_{2}\right|=\sqrt{1+81+4}
$$

$$
\therefore \cos \theta=\frac{\nabla \phi_{1} \cdot \nabla \phi_{2}}{1 \nabla \phi_{1}}=\sqrt{86}
$$

$$
\left|\nabla \phi_{1}\right|\left|\nabla \phi_{2}\right|
$$

$$
=\frac{(12 \overrightarrow{1}-8 \vec{j}-6 \vec{k}) \cdot(\vec{r}+9 \vec{j}-2 \vec{k})}{\sqrt{244}}
$$

$$
=\frac{12(1)-8(9)-6(-2)}{\sqrt{244} \sqrt{86}}
$$

$$
=\frac{-48}{2 \sqrt{61} \sqrt{86}}
$$

$$
\begin{aligned}
\cos \theta & =\frac{-24}{\sqrt{5946}} \\
\theta & =\cos ^{-1}\left[\frac{-24}{\sqrt{5246}}\right]
\end{aligned}
$$

<compat>ᄀ. Fard $a$ and D, such that the surfaces $a x^{2}-b y z=(a+2) x$ and $4 x^{2} y+z^{3}=4$ cut
extrogomally at $(1,-1,2)$
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DERIVATIVES: Gradient of a scalar field, Directional Derivative
sols.
Let $\phi_{1}=a x^{2}-b y z-(a+2)$

$$
\nabla \phi_{1}=\vec{r} \frac{\partial \phi_{1}}{\partial x}+\vec{\partial} \frac{\partial \phi_{1}}{\partial y}+\vec{k} \frac{\partial \phi}{\partial z}
$$

$$
\nabla \phi_{1}=\overrightarrow{1}[2 a x-(a+2)]+\vec{J}[-b z]+\overrightarrow{k^{\prime}}[-b y]
$$

$$
\nabla \phi_{1}(1,-1,2)=T[2 a-a-2]-2 b \rightarrow+b
$$

$$
=(a-2) \rightarrow-2 b \vec{j}+b \vec{k}
$$

$$
\text { and } \begin{aligned}
\phi_{2} & =4 x^{2} y+z^{3}-4 \\
\nabla \phi_{2} & =8 x y \rightarrow+4 x^{2} \vec{j}+3 z^{2} \vec{K} \\
\nabla \phi_{2}(1,-1,2) & =-8 r+4 j^{\prime}+12 \vec{F}
\end{aligned}
$$

Given two surfaces are cut orthogonally,

$$
\begin{gathered}
\text { ce, } \nabla \phi_{1} \cdot \nabla \phi_{2}=0 \\
{[(a-2) \rightarrow+2 b \vec{k}+b \cdot[-8 T+4 \vec{j}+12 \vec{k}]=0} \\
-8(a-2)-8 b+12 b=0 \\
-8 a+16-8 b+12 b=0 \\
-2 a+b+4=0
\end{gathered}
$$

ie, $2 a-b-4=0 \quad \longrightarrow(2)$
since $(1,-1,2)$ lies on the surface using (1).

$$
\begin{aligned}
& \phi_{1}(x, y, z)=0 \\
& a(1)^{2}-b(-1)(2)=(a+2)(1) \\
& a+2 b-a-2=0 \\
& 2 b=2 \Rightarrow 2 a=1 \\
& (2) \Rightarrow 2 a-1-4=0 \Rightarrow a=5 / 2
\end{aligned}
$$

