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## Gauss Seidel method :

(1) Solve by Gauss Seidel method :

$$x + y + 54z = 110$$
$$27x + 6y - 5z = 85$$
$$6x + 15y + 2z = 72.$$

Soln:

Let us rearrange the equations,

$$27x + 6y - 5z = 85 \rightarrow (1)$$
$$6x + 15y + 2z = 72 \rightarrow (2)$$
$$x + y + 54z = 110. \rightarrow (3)$$

$\therefore \ |27| > |6| + |5|$

$|15| > |6| + |2|$

$|54| > |1| + |1|.$

$$(1) \Rightarrow \quad x = \frac{85 - 6y + 5z}{27}$$

$$(2) \Rightarrow \quad y = \frac{72 - 6x - 2z}{15}$$

$$(3) \Rightarrow \quad z = \frac{110 - x - y}{54}.$$

Let $y = z = 0$

| Iteration No | $x = \frac{85-6y+5z}{27}$ | $y = \frac{72-6x-2z}{15}$ | $z = \frac{110-x-y}{54}$ |
|---|---|---|---|
| 1 | 3.148 | 3.541 | 1.913 |
| 2 | 2.432 | 3.572 | 1.926 |
| 3 | 2.426 | 3.573 | 1.926 |
| 4 | 2.425 | 3.573 | 1.926 |
| 5 | 2.425 | 3.573 | 1.926 |

Hence the soln is $x = 2.425$, $y = 3.573$, $z = 1.926$

$$z = \frac{1}{10}(24 - 0.9933 - 3\times 1.5070) = 1.8486$$

Fifth iteration:

$$x = \frac{1}{28}(32 - 4\times 1.5070 + 1.8486) = 0.9936$$

$$y = \frac{1}{17}(35 - 2\times 0.9936 - 4\times 1.8486) = 1.50696$$

$$z = \frac{1}{10}(24 - 0.9936 - 3\times 1.50696) = 1.8486$$

Sixth iteration:

$$x = \frac{1}{28}(32 - 4\times 1.50696 + 1.8486) = 0.9936$$

$$y = \frac{1}{17}(35 - 2\times 0.9936 - 4\times 1.848) = 1.50696$$

$$z = \frac{1}{10}(24 - 0.9936 - 3\times 1.50696) = 1.8486.$$

The solution is,

$$\boxed{x = 0.9936,\ y = 1.50696\ \&\ z = 1.8486}$$

(4) Solve the following system by Gauss-Seidel method:

$$9x - y + 2z = 9$$
$$x + 10y - 2z = 15$$
$$2x - 2y - 13z = -17$$

Solution:

The given system of equations are,

$$9x - y + 2z = 9 \longrightarrow (1)$$
$$x + 10y - 2z = 15 \longrightarrow (2)$$
$$2x - 2y - 13z = -17 \longrightarrow (3)$$

Clearly the coefficient matrix is diagonally dominant, we can apply Gauss-Seidel method.

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From ①, ② and ③, we have

$$x = \frac{1}{9}(9 + y - 2z) \rightarrow ④$$

$$y = \frac{1}{10}(15 - x + 2z) \rightarrow ⑤$$

$$z = \frac{1}{13}(2x - 2y + 17) \rightarrow ⑥$$

First iteration:

Putting $y = 0, z = 0$ in ④,

$x = 1$

Putting $x = 1, z = 0$ in ⑤,

$y = 1.4$

Putting $x = 1, y = 1.4$ in ⑥,

$z = 1.3626$

Second iteration:

Putting $y = 1.4, z = 1.3626$ in ④.

$$x = \frac{1}{9}(9 + 1.4 - 2 \times 1.3626) = 0.8528$$

Putting $x = 0.8528$, $z = 1.3626$ in ⑤,

$$y = \frac{1}{10}(15 - 0.8528 + 2 \times 1.3626) = 1.6872$$

Putting $x = 0.8528$, $y = 1.6872$ in ⑥.

$$z = \frac{1}{13}(2 \times 0.8528 - 2 \times 1.6872 + 17) = 1.1793$$

Third iteration:

$$x = \frac{1}{9}(9 + 1.6872 - 2 \times 1.1793) = 0.9254$$

$$y = \frac{1}{10}(15 - 0.9254 + 2 \times 1.1793) = 1.6433$$

$$z = \frac{1}{13}(2\times 0.9254 - 2\times 1.6433 + 17) = 1.1972$$

Fourth iteration:

$$x = \frac{1}{9}(9 + 1.6433 - 2\times 1.1972) = 0.9165$$

$$y = \frac{1}{10}(15 - 0.9165 + 2\times 1.1972) = 1.6478$$

$$z = \frac{1}{13}(2\times 0.9165 - 2\times 1.6478 + 17) = 1.1952$$

Fifth iteration:

$$x = \frac{1}{9}(9 + 1.6478 - 2\times 1.1952) = 0.9175$$

$$y = \frac{1}{10}(15 - 0.9175 + 2\times 1.1952) = 1.6473$$

$$z = \frac{1}{13}(2\times 0.9175 - 2\times 1.6473 + 17) = 1.1954$$

Sixth iteration:

$$x = \frac{1}{9}(9 + 1.6473 - 2\times 1.1954) = 0.9174$$

$$y = \frac{1}{10}(15 - 0.9174 + 2\times 1.1954) = 1.6473$$

$$z = \frac{1}{13}(2\times 0.9174 - 2\times 1.6473 + 17) = 1.1954$$

The solution is,

$$\boxed{x = 0.9174,\ y = 1.6473,\ z = 1.1954}$$