Simultaneous linear algebraic equations.

- Direct method
  - Gauss elimination method
  - Gauss Jordan method
- Indirect method
  - Gauss Jacobi method
  - Gauss Seidel method.

Gauss elimination method :

1. Solve $x+3y+3z=16$, $x+4y+3z=18$, $x+3y+4z=19$ by Gauss elimination method.

Solution: Given :

$$x+3y+3z=16$$
$$x+4y+3z=18$$
$$x+3y+4z=19$$

$$(A,B) = \left(\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 1 & 4 & 3 & 18 \\ 1 & 3 & 4 & 19 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

By Back Substitution method,

$\therefore$ $z = 3$

$y = 2$

$$x+3y+3z=16$$
$$x+3(2)+3(3)=16$$
$$x=1$$

$\therefore$ $\boxed{x=1,\ y=2,\ z=3}$

Gauss Jordan method :

1. Using Gauss Jordan method Solve the following equations : $10x+y+z=12$, $2x+10y+z=13$, $x+y+5z=7$.

Solution:

$$(A,B) = \left(\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 0 & -49 & -4 & -53 \\ 0 & -9 & -49 & -58 \end{array}\right) \quad \begin{array}{l} R_2 \to R_1 - 5R_2 \\ R_3 \to R_1 - 10R_3 \end{array}$$

$$= \left(\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 9 & 49 & 58 \end{array}\right) \quad \begin{array}{l} R_2/-1 \\ R_3/-1 \end{array}$$

$$= \left(\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & -2365 & -2365 \end{array}\right) \quad R_3 \to 9R_2 - 49R_3$$

$$= \left(\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad R_3/-2365$$

$$= \left(\begin{array}{ccc|c} -490 & 0 & -45 & -535 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad R_1 \to R_2 - 49R_1$$

$$= \left(\begin{array}{ccc|c} 490 & 0 & 45 & 535 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad R_1/-1$$

$$= \left(\begin{array}{ccc|c} 490 & 0 & 0 & 490 \\ 0 & 49 & 0 & 49 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad \begin{array}{l} R_1 \to R_1 - 45R_3 \\ R_2 \to R_2 - 4R_3 \end{array}$$

$\Rightarrow \quad z = 1$

$49y = 49 \Rightarrow y = 1$

$490x = 490 \Rightarrow x = 1$

$\therefore$ The solution is

$x = 1, \; y = 1, \; z = 1$