(1)

UNIT – III

Solution of equations & Eigen Value problems

Formulas :

(1) Newton Raphson method :

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Problems :

(1) Solve the equation $x^3 = 6x - 4$ using Newton's iterative method & correct to two decimal places.

Solution :

Given : $f(x) = x^3 - 6x + 4$, $f'(x) = 3x^2 - 6$

$f(0) = 0 - 0 + 4 = 4 = +ve$

$f(1) = 1 - 6 + 4 = -1 = -ve$

Hence the root lies between 0 and 1.

Let $x_0 = \frac{0+1}{2} = 0.5$

Formula : $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Put $n=0 \Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.71$

Put $n=1 \Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.71 - \frac{f(0.71)}{f'(0.71)} = 0.73$

Put $n=2 \Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.73 - \frac{f(0.73)}{f'(0.73)} = 0.73$

Hence the root is $\underline{0.73}$

(2) Find a root of $x \log_{10} x - 1.2 = 0$ by N.R method Correct to three decimal places.

Solution:

Given : $f(x) = x\log_{10} x - 1.2$; $f'(x) = \cancel{x}\cdot\frac{1}{\cancel{x}}\log_{10} e + \log_{10} x$

$= \log_{10} e + \log_{10} x$

$= 0.4343 + \log_{10} x$

$f(1) = \log_{10} 1 - 1.2 = -1.2 = -ve$

$f(2) = 2\log_{10} 2 - 1.2 = -0.598 = -ve$

$f(3) = 3\log_{10} 3 - 1.2 = 0.231 = +ve$

. Hence the root lies between 2 and 3.

Let $x_0 = \frac{2+3}{2} = 2.5$.

Formula : $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Put $n=0 \Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.5 - \frac{f(2.5)}{f'(2.5)} = 2.747$

Put $n=1 \Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.747 - \frac{f(2.747)}{f'(2.747)} = 2.741$

Put $n=2 \Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.741 - \frac{f(2.741)}{f'(2.741)} = 2.741$

Hence the root is 2.741

(3) Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to 6 places of decimals.

Soln:

Given : $f(x) = 3x - \cos x - 1$; $f'(x) = 3 + \sin x$

$f(0) = 0 - 1 - 1 = -2 = -ve$

$f(1) = 3 - \cos 1 - 1 = 1.459698 = +ve$.

Hence the root lies between 0 & 1.

Let $x_0 = \frac{0+1}{2} = 0.5$

Formula : $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

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Put $n=0 \Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.608519$

Put $n=1 \Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.608519 - \frac{f(0.608519)}{f'(0.608519)} = 0.607102$

Put $n=2 \Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.607102 - \frac{f(0.607102)}{f'(0.607102)} = 0.607102$

Hence the root is 0.607102.

(4) Find the iterative formula for finding the value of $\frac{1}{N}$ where N is a real number, using N-R method. Hence evaluate $\frac{1}{26}$ correct to 4 decimal places.

Solution:

Let $x = \frac{1}{N}$

$N = \frac{1}{x} \Rightarrow \frac{1}{x} - N = 0.$

$f(x) = \frac{1}{x} - N \quad ; \quad f'(x) = -\frac{1}{x^2}.$

Formula: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$= x_n - \frac{\left(\frac{1}{x_n} - N\right)}{-\frac{1}{x_n^2}} = x_n + x_n^2\left(\frac{1}{x_n} - N\right)$

$= x_n + x_n - Nx_n^2$

$\boxed{x_{n+1} = 2x_n - Nx_n^2}$ is the iterative formula. $\rightarrow$ (1)

To find 1/26:

Let $x_0 = 0.06$ (i.e., $1/26 = 0.038 \simeq 0.04$)

Here $N = 26$.

Put $n=0$ in (1) $\Rightarrow x_1 = 2x_0 - Nx_0^2 = 2(0.06) - 26(0.06)^2$

$= 0.0384$

Put $n=1$ in ① $\Rightarrow$ $x_2 = 2x_1 - Nx_1^2 = 2(0.0384) - 26(0.0384)^2$

$x_2 = 0.0385$

Put $n=2$ in ① $\Rightarrow$ $x_3 = 2x_2 - Nx_2^2 = 2(0.0385) - 26(0.0385)^2$

$x_3 = 0.0385.$

Hence the value of $\frac{1}{26} = 0.0385.$

⑤ Obtain Newton's iterative formula for finding $\sqrt{N}$ where N is a positive real number. Hence evaluate $\sqrt{142}$.

Solution:

Let $x = \sqrt{N}$

$x^2 = N$

$x^2 - N = 0$

$f(x) = x^2 - N \quad ; \quad f'(x) = 2x$

Formula: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$= x_n - \dfrac{x_n^2 - N}{2x_n} = \dfrac{2x_n^2 - x_n^2 - N}{2x_n}$

$\boxed{x_{n+1} = \dfrac{x_n^2 - N}{2x_n}}$ is the iterative formula. $\rightarrow$ ①

To find $\sqrt{142}$:

Let $x_0 = 12$.

Here $N = 142$.

Put $n=0$ in ①, $x_1 = \dfrac{x_0^2 - 142}{2x_0} = \dfrac{12^2 - 142}{2(12)} = 11.9167$

Put $n=1$ in ①, $x_2 = \dfrac{x_1^2 - 142}{2x_1} = \dfrac{11.9167^2 - 142}{2(11.9167)^2} = 11.9164$

Put $n=2$ in ①, $x_3 = \dfrac{x_2^2 - 142}{2x_2} = \dfrac{11.9164^2 - 142}{2(11.9164)^2} = 11.9164$

Hence the value of $\sqrt{142} = 11.9164$