Randomized block Design (RBD) (or Two way Classification

　　It is a two factor experiment.

Procedure :

Step 1 : Null hypothesis : H_0 : There is no significant difference between columns and rows.

　Alternative hypothesis : H_1 : There is a significant difference between columns and rows.

Step 2 : ＊ Find N
　　　　＊ Find T
　　　　＊ Find $C.F = T^2/N$

Step 3 : ＊ Find $SST = \sum x_1^2 + \sum x_2^2 + \cdots - C.F$

　　　　＊ Find $SSC = \dfrac{(\sum x_1)^2}{C_1} + \dfrac{(\sum x_2)^2}{C_2} + \cdots - C.F$

　　　　＊ Find $SSR = \dfrac{(\sum y_1)^2}{r_1} + \dfrac{(\sum y_2)^2}{r_2} + \cdots - C.F$

　　　　＊ Find $SSE = SST - SSC - SSR$

Step 4 : ANOVA table.

Source of Variation	Degree of freedom	Sum of Squares	Mean sum of squares	Variance ratio	Table value
Between Columns	$(c-1)$	SSC	$MSC = \dfrac{SSC}{C-1}$	$F_c = \dfrac{MSC}{MSE}$	$F_\alpha(c-1,\ (c-1)(r-1))$
Between rows	$(r-1)$	SSR	$MSR = \dfrac{SSR}{r-1}$	$F_R = \dfrac{MSR}{MSE}$	$F_\alpha(r-1,\ (c-1)(r-1))$
Between errors	$(c-1) \times (r-1)$	SSE	$MSE = \dfrac{SSE}{(r-1)(c-1)}$		

Step 5 : Decision : If $F_c < F_\alpha$, $F_R < F_\alpha$, H_0 is accepted otherwise it is rejected.

Problem :

① An experiment was designed to study the performance of 4 different detergents for cleaning of injectors. The following "cleanliness" readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines.

Detergent	Engine 1	Engine 2	Engine 3	Total
A	45	43	51	139
B	47	46	52	145
C	48	50	55	153
D	42	37	49	128
Total	182	176	207	565

Perform the ANOVA test at 0.01 level of significance whether there are differences in the detergents or in the engines.

Solution :

Fix origin = 50. Subtract each element from 50.

Engine \ Detergent	X_1	X_2	X_3	Total	X_1^2	X_2^2	X_3^2
A (y_1)	−5	−7	+1	−11	25	49	1
B (y_2)	−3	−4	2	−5	9	16	4
C (y_3)	−2	0	5	3	4	0	25
D (y_4)	−8	−13	−1	−22	64	169	1
Total	−18	−24	7	−35	102	234	31

Step 1: Null hypothesis H_0: There is no significant difference between engines and detergents.

Alternative hypothesis H_1: There is a significant difference between engines and detergents

Step 2:   ✶ $N = 12$

$$T = -35$$

$$C.F = T^2/N = 102.08$$

Step 3: $SST = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 - C.F$

$$= 102 + 234 + 31 - 102.08$$

$$\boxed{SST = 264.92}$$

$$SSC = \frac{(\Sigma x_1)^2}{C_1} + \frac{(\Sigma x_2)^2}{C_2} + \frac{(\Sigma x_3)^2}{C_3} - C.F$$

$$= \frac{(-18)^2}{4} + \frac{(-24)^2}{4} + \frac{7^2}{4} - 102.08$$

$$\boxed{SSC = 135.17}$$

$$SSR = \frac{(\Sigma y_1)^2}{r_1} + \frac{(\Sigma y_2)^2}{r_2} + \frac{(\Sigma y_3)^2}{r_3} + \frac{(\Sigma y_4)^2}{r_4} - C.F$$

$$= \frac{(-11)^2}{3} + \frac{(-5)^2}{3} + \frac{3^2}{3} + \frac{(-22)^2}{3} - 102.08$$

$$\boxed{SSR = 110.91}$$

$$SSE = TSS - SSC - SSR = 264.92 - 135.17 - 110.91$$

$$\boxed{SSE = 18.84}$$

Step 4: ANOVA table:

Source of Variation	Degree of freedom	Sum of Squares	Mean Sum of Squares	Variance ratio	Table value at 1% level
Between Columns	$C-1 = 3-1$ $= 2$	$SSC = 135.17$	$MSC = \frac{SSC}{C-1}$ $= 67.585$	$F_c = \frac{MSC}{MSE}$ $= 21.52$	$F_\alpha(2,6)$ $= 10.92$
Between rows	$r-1 = 4-1$ $= 3$	$SSR = 110.91$	$MSR = \frac{SSR}{r-1}$ $= 36.97$	$F_R = \frac{MSR}{MSE}$ $= 11.77$	$F_\alpha(3,6)$ $= 9.78$
Between errors	$(c-1)(r-1)$ $= 6$	$SSE = 18.84$	$MSE = \frac{SSE}{(c-1)(r-1)} = 3.14$		

Step 5: **Decision:** Since $F_c > F_\alpha$ and $F_R > F_\alpha$, H_0 is Rejected

∴ There is a significant difference between engines and detergents

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