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#### **DEPARTMENT OF MATHEMATICS**



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Test 2:

Test of significance for the difference between two Population means when population standard deviations are not known:

Let  $\overline{x}_1$  and  $\overline{x}_2$  are the means of two independent samples of sizes n, and n<sub>2</sub> from a normal population with mean  $\mu_1$ , and  $\mu_2$  and standard deviation  $-s_1$  and  $-s_2$ .

We want to test whether the mean  $\mu_i$ and  $\mu_2$  of the two populations are equal or not Under  $H_0$ :  $\mu_i = \mu_2$  the test-Statistic is defined as,

$$t = \overline{x_1 - x_2}$$

$$S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

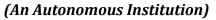
where  $S^2 = n_1 s_1^2 + n_2 s_2^2$  $n_1 + n_2 - 2$ 

with  $V = n_1 + n_2 - 2$  degrees of freedom.

Note :

Suppose the Samples Sizes are equal. i.e.,  $n_1 = n_2 = n$ . Then we have n pair of values. Further we assume that the n pairs are independent. Then





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the test-Statistic will be,

$$t = \overline{x_1 - \overline{x_2}}$$

$$\int \frac{S_1^2 + S_2^2}{n - 1}$$

with V = n + n - 2 = 2n - 2 degrees of freedom.





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### Problems :

(1) Two salesmen A and B are working in a Certain district. From a sample Survey conducted by the Head office, the following results were obtained. State whether there is any significant difference in the average sales between the two sales men:

	A	В
Number of Sales	20	18
Average Sales (in Rs.)	170	205
Standard Deviation (in Rs.)	20	25

Solution :

Given:  $n_1 = 20$ ,  $n_2 = 18$  $\overline{\chi}_1 = 170$ ,  $\overline{\chi}_2 = 205$  $S_1 = 20$ ,  $S_2 = 25$ .

Null hypothesis :  $H_0$  : There is no significant difference in the average sales of the two sales men

Alternative hypothesis :  $H_1 : \mu_1 \neq \mu_2$ 

Test-Statistic :

$$t = \frac{\overline{x_1 - x_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

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where 
$$S^{2} = \frac{n_{1} \cdot s_{1}^{2} + n_{2} \cdot s_{2}^{2}}{n_{1} + n_{2} - \lambda}$$
  
 $S^{2} = \frac{\lambda_{0} \times \lambda_{0}^{2} + 18 \times 25^{2}}{\lambda_{0} + 18 - \lambda} = \frac{19250}{36}$   
 $S^{2} = 534 \cdot 7\lambda$   
 $S = 23 \cdot 12$   
 $t = \frac{170 - \lambda_{0}5}{\lambda_{3} \cdot 1\lambda}$   
 $= \frac{-35}{\lambda_{3} \cdot 1\lambda} = -4 \cdot 65$   
 $1t_{1} = 4 \cdot 65$ 

Table value :

At  $\alpha = 1$  / Los,  $V = n_1 + n_2 - 2 = 36$  d.o.f, the table value of t is given by,

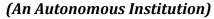
$$t_{d} = 2.58$$

Decision :

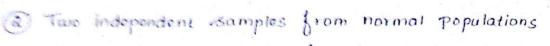
Since ItI > ta, Ho is rejected.

Hence the two salesmen differ Significantly with regard to their average sales





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with equal variance gave the following :

Sample	Size	Mean	S.D
1	16	23.4	2.5
2	12	24.9	2.8

Is the difference between the means significant ? Solution:

Griven: 
$$n_1 = 16$$
,  $n_2 = 12$   
 $\overline{x_1} = 23.4$ ,  $\overline{x_2} = 24.9$   
 $s_1 = 2.5$ ,  $s_2 = 2.8$ 

Null hypothesis:  $H_0$ : There is no significant difference between the means i.e.,  $H_0$ :  $\mu_1 = \mu_2$ 

Alternative hypothesis :  $H_1 = \mu_1 \neq \mu_2$  (Two-tailed test)

Test - Statistic :

$$t = \frac{\overline{x_{1} - \overline{x_{2}}}}{S \sqrt{\frac{1}{n_{1}} + \frac{1}{n_{2}}}}$$
where  $S^{2} = \frac{n_{1} - s_{1}^{2} + n_{2} - s_{2}^{2}}{n_{1} + n_{2} - 2}$ 

$$S^{2} = \frac{16 \times 2 \cdot 5^{2} + 12 \times 2 \cdot 8^{2}}{16 + 12 - 2}$$

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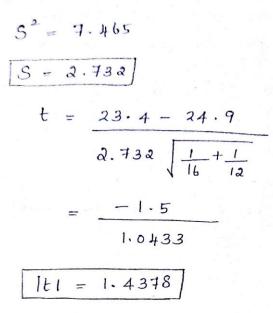


Table value :

At  $\alpha = 5$  % Los,  $V = n_1 + n_2 - 2 = 16 + 12 - 2$ V = 26 d.o.f, the table value of t is given by,

$$t_{\chi} = 2.06$$

Decision :

Since  $|t| \ge t_{\alpha}$ ,  $H_{\delta}$  is accepted. .: The difference is not significant.

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(115) Does it show superiority of diet A over diet B? Solution : Griven:  $n_1 = 10$  $h_{2} = 8$ Null hypothesis : Ho : There is no significant difference in increase of weights . i.e.,  $H_0: \mu_1 = \mu_2$ Alternative hypothesis:  $H_1$ :  $\mu_1 = \mu_2$  (Right tailed test) Calculation of sample means and Sample S. D's:  $\overline{x}_1 = \underline{x}_1 = \underline{b}_1 = \underline{b}_1 = \underline{b}_1$  $\overline{\chi_2} = \frac{\leq \chi_2}{n} = \frac{40}{a} = 5$  $S_{1}^{2} = \frac{5\chi_{1}^{2}}{n} - \left(\frac{5\chi_{1}}{n}\right)^{2} = \frac{512}{10} - \left(6\cdot4\right)^{2} = 10.24$  $-S_{2}^{2} = \frac{\leq \chi_{2}^{2}}{n} - \left(\frac{\leq \chi_{2}}{n}\right)^{2} = \frac{282}{n} - (5)^{2} = 10.25$ Test - Statistic :  $t = \overline{x_1} - \overline{x_2}$  $S\left[\frac{1}{n}+\frac{1}{n}\right]$ where  $S^2 = n_1 S_1^2 + h_2 S_2^2$  $n_{1} + n_{2} - 2$ 

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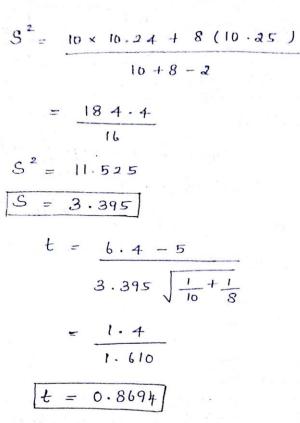


Table value :

At  $\alpha = 5$  / Los,  $V - n_1 + n_2 - 2 = 16$  d.o.f, the table value of t is given by,

 $t_{d} = 1.75$ 

Decision :

Since  $t \ge t_{\alpha}$ ,  $H_{0}$  is accepted. Hence we Cannot Conclude that diet A is superior to diet B.

