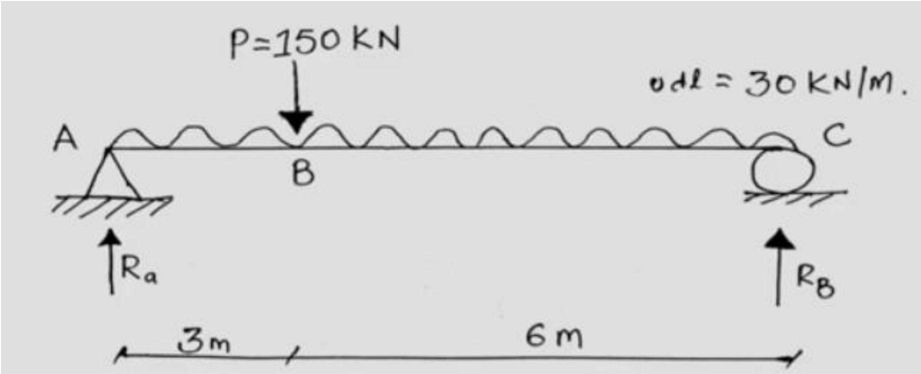
Castigliano's theorems and their applications



Determine the vertical deflection at point 150KN load in fig.
Take E: 232*10^6 KN/m^2 and I= 8.789*10^-4 m^4.

$$\Delta_B = \frac{1}{EI}\int_0^L M\left(\frac{\partial M}{\partial P}\right)dx$$

This is deflection formula for castigliano therorem.Hence, our main consideration is to calculate : M & $\frac{\partial M}{\partial P}$

$\sum Mc = 0$

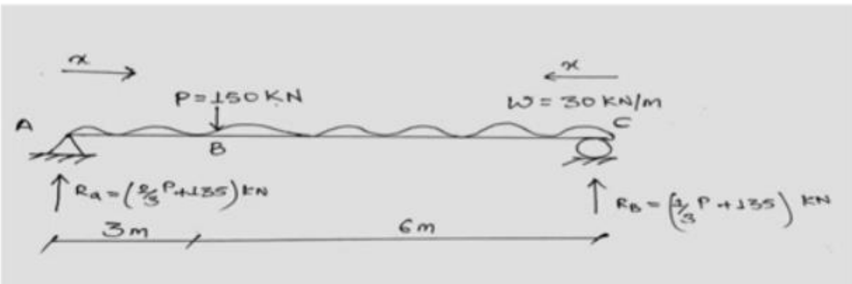
or, 9Ra=6P+30*9*$\frac{9}{2}$

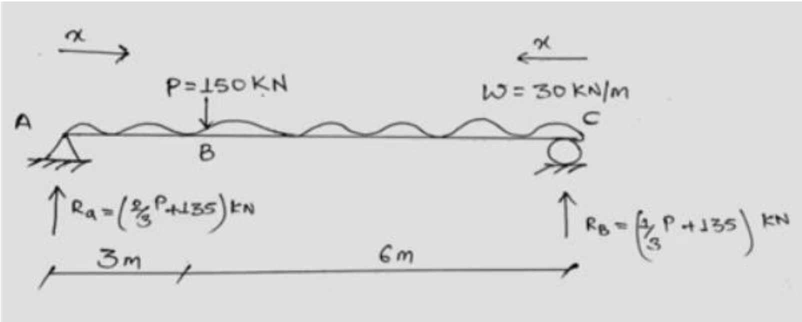
or, Ra= $\frac{2P}{3}$ + 135 KN

$\sum Ma = 0$

or, 9Rc=3P+30*9*$\frac{9}{2}$

or, Rc= $\frac{P}{3}$ + 135 KN





Segment	Origin	limits	Moment	$\frac{\partial M}{\partial P}$
AB	A	0 to 3m	$\frac{2}{3}Px + 135x - \frac{30x^2}{2}$	$\frac{2}{3}x$
CB	C	0 to 6	$\frac{P}{3}x + 135x - \frac{30x^2}{2}$	$\frac{1}{3}x$

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or, $\Delta_B = \frac{1}{EI}[\int_0^3 (\frac{2P}{3}x + 135x - 15x^2)(\frac{2}{3}x)\,dx + \int_0^6 (\frac{P}{3}x + 135x - 15x^2)\left(\frac{1}{3}x\right)dx\,]$

substitute P=150KN and upon integration, we get :

$$\Delta_B = 0.01975\text{m or } 19.75\text{mm}$$

i.e The point B deflects 19.75mm downward due to point load 150KN