UNIT - 1 STRESS, STRAIN DEFORMATION OF SOLIDS

VOLUMETRIC STRAIN

Whenever a body is subjected to a single force (or a system of forces), it undergoes some changes in its dimensions. A little consideration will show, that the change in dimensions of a body will cause some changes in its volume. The ratio of change in volume, to the original volume, is known as volumetric strain. Mathematically volumetric strain,

$$\varepsilon_V = \delta V / V$$

where δV = Change in volume, and

V = Original volume.

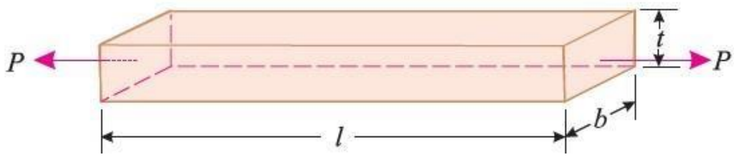
Though there are numerous ways, in which a force (or a system of forces) may act, yet the following are important from the subject point of view:

1. A rectangular body subjected to an axial force.

2. A rectangular body subjected to three mutually perpendicular forces.

Now we shall discuss the volumetric strains on all the types of bodies one by one in the following pages:

VOLUMETRIC STRAIN OF A RECTANGULAR BODY SUBJECTED TO AN AXIAL FORCE



Volumetric strain

volumetric strain,

$$\frac{\delta V}{V} = \frac{V \times \frac{P}{btE}\left(1-\frac{2}{m}\right)}{V} = \frac{P}{btE}\left(1-\frac{2}{m}\right)$$

$$= \varepsilon\left(1-\frac{2}{m}\right) \qquad \ldots\left(\because \frac{P}{btE} = \varepsilon = \text{Strain}\right)$$

NOTE. The above formula holds good for compressive force also.

A steel bar 2 m long, 20 mm wide and 15 mm thick is subjected to a tensile load of 30 kN. Find the increase in volume, if Poisson's ratio is 0.25 and Young's modulus is 200 GPa.

Given Data: Length (l) = 2 m = 2 × 10^3 mm; Width (b) = 20 mm; Thickness (t) = 15 mm; Tensile load (P) = 30 kN = 30 × 103 N; Poisson's ratio $1/m$ = 0.25 or m = 4 and Young's modulus of elasticity (E) = 200 GPa = 200 × 10^3 N/mm^2.

Let δV = Increase in volume of the bar.

We know that original volume of the bar,

$V = l.b.t = (2 \times 103) \times 20 \times 15 = 600 \times 10^3$ mm^3

and

$$\frac{\delta V}{V} = \frac{P}{btE}\left(1-\frac{2}{m}\right) = \frac{30\times10^3}{20\times15\times(200\times10^3)}\left(1-\frac{2}{4}\right) = 0.000\ 25$$

$\therefore$ $\delta V = 0.000\ 25 \times V = 0.000\ 25 \times (600 \times 10^3) = 150$ mm^3 **Ans.**

A copper bar 250 mm long and 50 mm × 50 mm in cross-section is subjected to an axial pull in the direction of its length. If the increase in volume of the bar is 37.5 mm^3, find the magnitude of the pull. Take m = 4 and E = 100 GPa.

Given Data: Length (l) = 250 mm ; Width (b) = 50 mm ; Thickness (t) = 50 mm ; Increase in volume (δV) = 37.5 mm3 ; (m) = 4 and

modulus of elasticity (E) = 100 GPa = 100×10^3 N/mm^2.

Let P = Magnitude of the pull in kN.

We know that

We know that original volume of the copper bar,

$$V = l.b.t = (250 \times 50 \times 50) = 625 \times 10^3 \text{ mm}^3$$

and

$$\frac{\delta V}{V} = \frac{P}{btE}\left(1-\frac{2}{m}\right) = \frac{P}{50\times10\times(100\times10^3)}\left(1=\frac{2}{4}\right)$$

or

$$\frac{37.5}{625\times10^3} = \frac{P}{500\times10^6}$$

$\therefore$

$$P = \frac{37.5\times(500\times10^6)}{625\times10^3} = 30\times10^3 \text{ N} = 30 \text{ kN} \quad \textbf{Ans.}$$

A steel bar 50 mm × 50 mm in cross-section is 1.2 m long. It is subjected to an axial pull of 200 kN. What are the changes in length, width and volume of the bar, if the value of Poisson's ratio is 0.3? Take E as 200 GPa.

Given Data: Width (b) = 50 mm; Thickness (t) = 50 mm ; Length (l) = 1.2 m = 1.2×10^3 mm ; Axial pull (P) = 200 kN = 200×10^3 N; Poisson's ratio (1 / m) = 0.3 and

Modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2.

Change in length

We know that change in length,

$$\delta l = \frac{Pl}{AE} = \frac{200\times10^3\times(1.2\times10^3)}{(50\times50)\times(200\times10^3)} = 0.48 \text{ mm} \quad \textbf{Ans.}$$

Change in width

We know that linear strain,

$$\varepsilon = \frac{\delta l}{l} = \frac{0.48}{1.2 \times 10^3} = 0.0004$$

and lateral strain $= \frac{1}{m} \times \varepsilon = 0.3 \times 0.0004 = 0.000\ 12$

$\therefore$ Change in width, $\delta b = b \times \text{Lateral strain} = 50 \times 0.000\ 12 = 0.006$ mm **Ans.**

Change in volume

We also know that volume of the bar,

$$V = l.b.t = (1.2 \times 10^3) \times 50 \times 50 = 3 \times 10^6 \text{ mm}^3$$

and

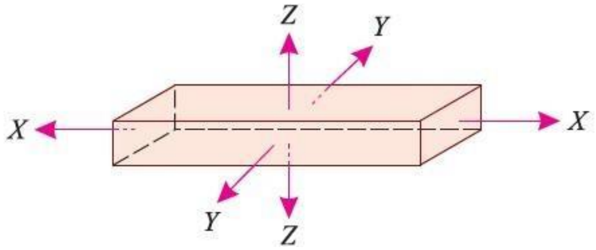
$$\frac{\delta V}{V} = \frac{P}{btE}\left(1 - \frac{2}{m}\right) = \frac{200 \times 10^3}{50 \times 50 \times (200 \times 10^3)}[1 - (2 \times 0.3)]$$

$$= 0.000\ 16$$

$\therefore$ $\delta V = 0.000\ 16\ V = 0.00016 \times (3 \times 10^6) = 480 \text{ mm}^3$ **Ans.**

VOLUMETRIC STRAIN OF A RECTANGULAR BODY SUBJECTED TO THREE MUTUALLY PERPENDICULAR FORCES

Consider a rectangular body subjected to direct tensile stresses along three mutually perpendicular axes as shown in Figure.



Volumetric strain

Let σ_x = Stress in *x-x* direction,

σ_y = Stress in *y-y* direction,

σ_z = Stress in *z-z* direction and

E = Young's modulus of elasticity.

$\therefore$ Strain in *x-x* direction due to stress σ_x,

$$\varepsilon_x = \frac{\sigma_x}{E}$$

Similarly, $\varepsilon_y = \frac{\sigma_y}{E}$ and $\varepsilon_z = \frac{\sigma_z}{E}$

The resulting strains in the three directions, may be found out by the principle of superposition, *i.e.*, by adding algebraically, the strains in each direction due to each individual stress. For the three tensile stresses shown in above Figure. (Taking tensile strains as +ve and compressive strains as –ve) the resultant strain in *x-x* direction,

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\sigma_y}{mE} - \frac{\sigma_z}{mE} = \frac{1}{E}\left[\sigma_x - \frac{\sigma_y}{m} - \frac{\sigma_z}{m}\right]$$

Similarly,

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_z}{mE} = \frac{1}{E}\left[\sigma_y - \frac{\sigma_x}{m} - \frac{\sigma_z}{m}\right]$$

and

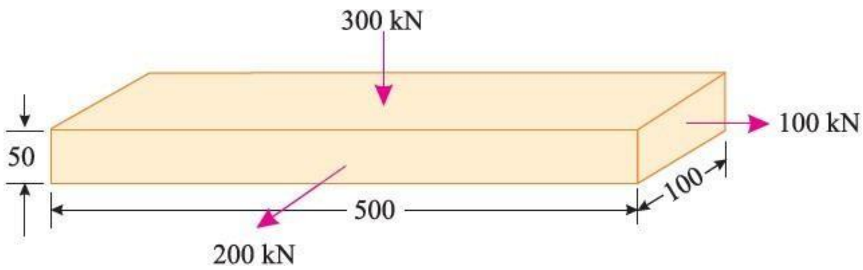
$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_y}{mE} = \frac{1}{E}\left[\sigma_z - \frac{\sigma_x}{m} - \frac{\sigma_y}{m}\right]$$

The volumetric strain may then be found by the relation;

$$\frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

NOTE. In the above relation, the values of εx, εy and εz should be taken tensile as positive and compressive as negative.

A rectangular bar 500 mm long and 100 mm × 50 mm in cross-section is subjected to forces as shown in Figure. What is the change in the volume of the bar? Take modulus of elasticity for the bar material as 200 GPa and Poisson's ratio as 0.25.



Given Data: Length (l) = 500 mm; Width (b) = 100 mm; Thickness (t) = 50 mm;

Force in x-direction (P_x) = 100 kN = 100×10^3 N (Tension);

Force in y-direction (P_y) = 200 kN = 200×10^3N (Tension);

Force in z-direction (P_z) = 300 kN = 300×10^3 N (Compression);

Modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2 and

Poisson's ratio ($1/m$) = 0.25 or m = 4.

Let δV = Change in the volume of the bar.

We know that original volume of the rectangular bar,

$V = l \times b \times t$ = 500 × 100 × 50 = 2.5 × 106 mm3 and stress in *x-x* direction,

$$\sigma_x = \frac{P_x}{A_x} = \frac{100 \times 10^3}{100 \times 50} = 20 \text{ N/mm}^2 \text{ (Tension)}$$

Similarly,
$$\sigma_y = \frac{P_y}{A_y} = \frac{200 \times 10^3}{500 \times 50} = 8 \text{ N/mm}^3 \text{ (Tension)}$$

and
$$\sigma_z = \frac{P_z}{A_z} = \frac{300 \times 10^3}{500 \times 100} = 6 \text{ N/mm}^2 \text{ (Compression)}$$

We also know that resultant strain in *x*-*x* direction considering tension as positive and compression as negative

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\sigma_y}{mE} + \frac{\sigma_z}{mE} = +\frac{20}{E} - \frac{8}{4E} + \frac{6}{4E} = \frac{19.5}{E}$$

Similarly
$$\varepsilon_y = +\frac{\sigma_y}{E} - \frac{\sigma_x}{mE} + \frac{\sigma_z}{mE} = +\frac{8}{E} - \frac{20}{4E} + \frac{6}{4E} = \frac{4.5}{E}$$

and
$$\varepsilon_z = -\frac{\sigma_z}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_y}{mE} = -\frac{6}{E} - \frac{20}{4E} - \frac{8}{4E} = -\frac{13}{E}$$

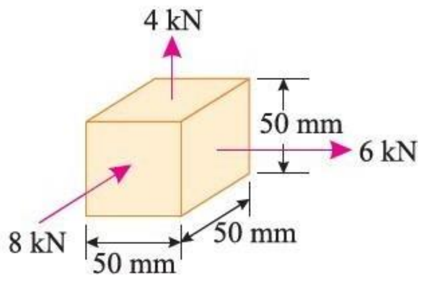
We also know that volumetric strain,

$$\frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\frac{\delta V}{2.5 \times 10^6} = \frac{19.5}{E} + \frac{4.5}{E} - \frac{13}{E} = \frac{11}{E} = \frac{11}{200 \times 10^3} = 0.055 \times 10^{-3}$$

$$\therefore \quad \delta V = (0.055 \times 10^{-3}) \times (2.5 \times 10^6) = 137.5 \text{ mm}^3 \quad \textbf{Ans.}$$

A steel cube block of 50 mm side is subjected to a force of 6 kN (Tension), 8 kN (Compression) and 4 kN (Tension) along x, y and z direction respectively. Determine the change in volume of the block. Take E as 200 GPa and m as 10/3.



Given Data: Side of the cube = 50 mm; Force in x direction

(P_x) = 6 kN = 6 × 10^3 N (Tension); Force in y-direction

(P_y) = 8 kN = 8 × 10^3 N (Compression); Force in z-direction

(P_z)= 4 kN = 4 × 10^3 N (Tension) and

Modulus of elasticity (E) = 200 GPa = 200 × 10^3 N/mm^2 and m = 10 / 3

Let δV = Change in volume of the block.

We know that original volume of the steel cube,

$$V = 50 \times 50 \times 50 = 125 \times 10^3 \text{ mm}^3$$

and stress in x-x direction,

$$\sigma_x = \frac{P_x}{A} = \frac{6 \times 10^3}{2500} = 2.4 \text{ N/mm}^2 \text{ (Tension)}$$

Similarly $$\sigma_y = \frac{P_y}{A} = \frac{8 \times 10^3}{2500} = 3.2 \text{ N/mm}^2 \text{ (Compression)}$$

and $$\sigma_z = \frac{P_z}{A} = \frac{4 \times 10^3}{2500} = 1.6 \text{ N/mm}^2 \text{ (Tension)}$$

We also know that resultant strain in x-x direction considering tension as positive and compression as negative,

$$\varepsilon_x = \frac{\sigma_x}{E} + \frac{\sigma_y}{mE} - \frac{\sigma_z}{mE} = \frac{2.4}{E} + \frac{3.2 \times 3}{10E} - \frac{1.6 \times 3}{10E} = \frac{2.88}{E}$$

Similarly, $$\varepsilon_y = -\frac{\sigma_y}{E} - \frac{\sigma_x}{mE} - \frac{\sigma_z}{mE} = -\frac{3.2}{E} - \frac{2.4 \times 3}{10E} - \frac{1.6 \times 3}{10E} = -\frac{4.4}{E}$$

and $$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\sigma_x}{mE} + \frac{\sigma_y}{mE} = \frac{1.6}{E} - \frac{2.4 \times 3}{10E} + \frac{3.2 \times 3}{10E} = \frac{1.84}{E}$$

We also know that volumetric strain,

$$\frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\frac{\delta V}{125 \times 10^3} = \frac{2.88}{E} - \frac{4.4}{E} + \frac{1.84}{E} = \frac{0.32}{E} = \frac{0.32}{200 \times 10^3}$$

$$\therefore \quad \delta V = 125 \times 10^3 \times \frac{0.32}{200 \times 10^3} = 0.2 \text{ mm}^3$$ **Ans.**

EXERCISE

1. A steel bar 1.2 m long, 50 mm wide and 40 mm thick is subjected to an axial pull of 150 kN in the direction of its length. Determine the change in volume of the bar. Take *E* = 200 GPa and m = 4. [**Ans.** 450 mm3]

2. A steel block 200 mm × 20 mm × 20 mm is subjected to a tensile load of 40 kN in the direction of its length. Determine the change in volume, if *E* is 205 GPa and 1/m = 0.3. [**Ans.** 15.6 mm3]