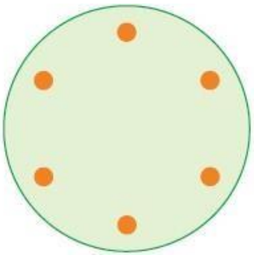
UNIT - 1 STRESS, STRAIN DEFORMATION OF SOLIDS

STRESSES IN THE BARS OF COMPOSITE STRUCTURES

A bar made up of two or more different materials, joined together is called a composite bar. The bars are joined in such a manner, that the system extends or contracts as one unit, equally, when subjected to tension or compression. Following two points should always be kept in view, while solving example on composite bars:

9. A reinforced concrete circular section of 50,000 mm2 cross-sectional area carries 6 reinforcing bars whose total area is 500 mm2. Find the safe load, the column can carry, if the concrete is not to be stressed more than 3.5 MPa. Take modular ratio for steel and concrete as 18.



Given Data: Area of column = 50,000 mm^2; No. of reinforcing bars = 6; Total area of steel bars (A_S) = 500 mm^2; Max stress in concrete (σ_C) = 3.5 MPa = 3.5 N/mm^2 and modular ratio (E_S / E_C) = 18.

We know that area of concrete,

A_C = 50,000 – 500 = 49,500 mm^2

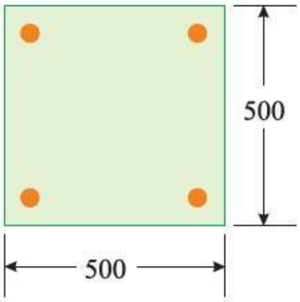
and stress in steel,

$$\sigma_S = \frac{E_S}{E_C} \times \sigma_C = 18 \times 3.5 = 63 \text{ N/mm}^2$$

∴ Safe load, the column can carry,

$$P = (\sigma_S \cdot A_S) + (\sigma_C \cdot A_C) = (63 \times 500) + (3.5 \times 49{,}500) \text{ N}$$
$$= 204\,750 \text{ N} = 204.75 \text{ kN} \qquad \textbf{Ans.}$$

10. A reinforced concrete column 500 mm × 500 mm in section is reinforced with 4 steel bars of 25 mm diameter, one in each corner. The column is carrying a load of 1000 kN. Find the stresses in the concrete and steel bars. Take E for steel = 210 GPa and E for concrete = 14 GPa.



Given Data: Area of column = 500 × 500 = 2,50,000 mm^2; No. of steel bars (n) = 4;

Diameter of steel bars (d) = 25 mm ; Load on column (P) = 1,000 kN = 1,000 × 10^3 N ;

Modulus of elasticity of steel (E_S) = 210 GPa and

Modulus of elasticity of concrete (E_C) = 14 GPa.

Let σ_S = Stress in steel, and

σ_C = Stress in concrete.

We know that area of steel bars,

$A_S = 4 \times \pi/4 \times (d)^2$ mm^2 ...(i)

$$= 4 \times \frac{\pi}{4} \times (25)^2 = 1963 \text{ mm}^2$$

$\therefore$ Area of concrete, $A_C = 250,000 - 1963 \text{ mm}^2 = 248\,037 \text{ mm}^2$

We also know that stress in steel,

$$\sigma_S = \frac{E_S}{E_C} \times \sigma_C = \frac{210}{14} \times \sigma_C = 15\,\sigma_C \quad ...(ii)$$

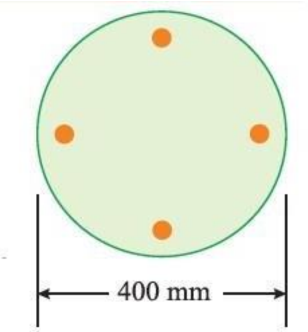
and total load (P), $1{,}000 \times 10^3 = (\sigma_S \cdot A_S) + (\sigma_C \cdot A_C)$

$$= (15\,\sigma_C \times 1963) + (\sigma_C \times 248\,037) = 277\,482\,\sigma_C$$

$$\sigma_C = \frac{1{,}000 \times 10^3}{277\,482} = 3.6 \text{ N/mm}^2 = 3.6 \text{ MPa} \quad \textbf{Ans.}$$

and $\sigma_S = 15\,\sigma_C = 15 \times 3.6 = 54$ MPa **Ans.**

11.A reinforced concrete circular column of 400 mm diameter has 4 steel bars of 20 mm diameter embedded in it. Find the maximum load which the column can carry, if the stresses in steel and concrete are not to exceed 120 MPa and 5 MPa respectively. Take modulus of elasticity of steel as 18 times that of concrete.



Given Data: Diameter of column (D) = 400 mm; No. of reinforcing bars = 4; Diameter of bars (d) = 20 mm; Maximum stress in steel($\sigma_S(max)$) = 120 MPa = 120 N/mm^2;

Maximum stress in concrete ($\sigma_C(max)$) = 5 MPa = 5 N/mm^2 and modulus of elasticity of steel (E_S) = 18 E_C.

We know that total area of the circular column.

$$= \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (400)^2 = 125\,660 \text{ mm}^2$$

and area of reinforcement (*i.e.*, steel),

$$A_S = 4 \times \frac{\pi}{4} \times (d)^2 = 4 \times \frac{\pi}{4} \times (20)^2 \text{ mm}^2 = 1257 \text{ mm}^2$$

∴ Area of concrete,

$$A_C = 125\,660 - 1257 = 124\,403 \text{ mm}^2$$

First of all let us find out the maximum stresses developed in the steel and concrete. We know that if the stress in steel is 120 N/mm^2, then stress in the concrete.

$$\sigma_C = \frac{E_C}{E_S} \times \sigma_S = \frac{1}{18} \times 120 = 6.67 \text{ N/mm}^2 \quad ...(i)$$

It is more than the stress in the concrete (*i.e.*, 5 N/mm^2). Thus these stresses are not accepted. Now if the stress in concrete is 5 N/mm^2, then stress in steel,

$$\sigma_S = \frac{E_S}{E_C} \times \sigma_C = 18 \times 5 = 90 \text{ N/mm}^2 \quad ...(ii)$$

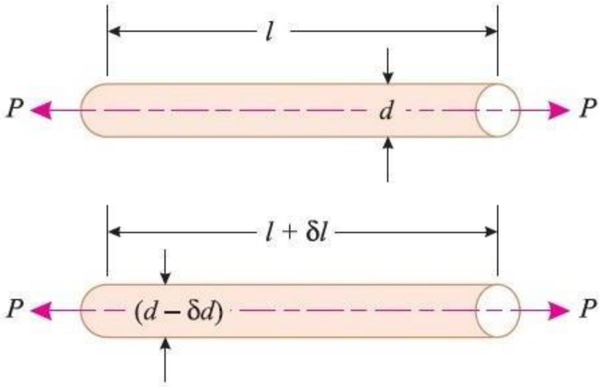
It is less than the stress is steel (*i.e.*, 120 N/mm^2). It is thus obvious that stresses in concrete and steel will be taken as 5 N/mm^2 and 90 N/mm^2 respectively. Therefore maximum load, which the column can carry.

$$P = (\sigma_C \,.\, A_C) + (\sigma_S \,.\, A_S) = (5 \times 124\,403) + (90 \times 1257) \text{ N}$$
$$= 735\,150 \text{ N} = 735.15 \text{ kN} \quad \textbf{Ans.}$$

PRIMARY OR LINEAR STRAIN

Figures showing Linear and Lateral strain

Whenever some external force acts on a body, it undergoes some deformation. Now consider a circular bar subjected to a tensile force as shown in Figure.



Let l = Length of the bar,

d = Diameter of the bar,

P = Tensile force acting on the bar, and

$d\,l$ = Increase in the length of the bar, as a result of the tensile force.

The deformation of the bar per unit length in the direction of the force, *i.e.*, $\vartheta l/l$ is known as primary or linear strain.

SECONDARY OR LATERAL STRAIN

We have already discussed in the last article the linear deformation of a circular bar of length l and diameter d subjected to a tensile force P. If we actually study the deformation of the bar, we will find that bar has extended through a length ϑl, which will be followed by the decrease of diameter from d to $(d - \vartheta d)$ as shown in above Figure. Similarly, if the bar is subjected to a compressive force, the length of the bar will decrease by ϑl which will be followed by the increase of diameter from d to $(d + \vartheta d)$. It is thus obvious that every direct stress is always accompanied by a strain in its own direction and an opposite kind of strain in every direction at right angles to it. Such a strain is known as secondary or lateral strain.

POISSON'S RATIO

It has been experimentally found, that if a body is stressed within its elastic limit, the lateral strain bears a constant ratio to the linear strain. Mathematically:

Lateral strain / Linear strain = (constant)

This constant is known as Poisson's ratio and is denoted by

1 / m or μ. Mathematically,

Lateral strain = $1 / m \times \varepsilon = \mu\, \varepsilon$

The value of Poisson's ratio of materials, in every day use, are given below :

S. No.	Material	Poisson's ratio $\left(\frac{1}{m} \text{ or } \mu\right)$		
1.	Steel	0.25	to	0.33
2.	Cast iron	0.23	to	0.27
3.	Copper	0.31	to	0.34
4.	Brass	0.32	to	0.42
5.	Aluminium	0.32	to	0.36
6.	Concrete	0.08	to	0.18
7.	Rubber	0.45	to	0.50

A steel bar 2 m long, 40 mm wide and 20 mm thick is subjected to an axial pull of 160 kN in the direction of its length. Find the changes in length, width and thickness of the bar. Take E = 200 GPa and Poisson's ratio = 0.3.

Given Data: Length (l) = 2 m = 2×10^3 mm; Width (b) = 40 mm; Thickness (t) = 20 mm; Axial pull (P) = 160 kN = 160×10^3 N; Modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2 poison's ratio ($1 / m$) = 0.3.

We know that change in length,

Change in length

We know that change in length,

$$\delta l = \frac{Pl}{AE} = \frac{(160 \times 10^3) \times (2 \times 10^3)}{(40 \times 20) \times (200 \times 10^3)} = 2 \text{ mm} \qquad \textbf{Ans.}$$

Change in width

We know that linear strain,

$$\varepsilon = \frac{\delta l}{l} = \frac{2}{2 \times 10^3} = 0.001$$

and lateral strain

$$= \frac{1}{m} \times \varepsilon = 0.3 \times 0.01 = 0.0003$$

∴ Change in width,

$$\delta b = b \times \text{Lateral strain} = 40 \times 0.0003 = 0.012 \text{ mm} \qquad \textbf{Ans.}$$

Change in thickness

We also know that change in thickness,

$$\delta t = t \times \text{Lateral strain} = 20 \times 0.0003 = 0.006 \text{ mm} \qquad \textbf{Ans.}$$

A metal bar 50 mm × 50 mm in section is subjected to an axial compressive load of 500 kN. If the contraction of a 200 mm gauge length was found to be 0.5 mm and the increase in thickness 0.04 mm, find the values of Young's modulus and Poisson's ratio for the bar material.

Given Data: Width (b) = 50 mm ; Thickness (t) = 50 mm ; Axial compressive load (P) = 500 kN = 500×10^3 N ; Length (l) = 200 mm ; Change in length (δl) = 0.5 mm and change in thickness (δt) = 0.04 mm.

Value of Young's modulus for the bar material

Let E = Value of Young's modulus for the bar material.

We know that contraction of the bar (δl),

$$0.5 = \frac{P.l}{A.E} = \frac{(500 \times 10^3) \times 200}{(50 \times 50) \times E} = \frac{40 \times 10^3}{E}$$

$$\therefore \quad E = \frac{40 \times 10^3}{0.5} = 80 \times 10^3 \text{ N/mm}^2 = 80 \text{ GPa}$$

Value of Poisson's ratio for the bar material

Let $\frac{1}{m}$ = Value of Poisson's ratio for the bar material.

We know that linear strain,

and lateral strain $= \frac{1}{m} \times \text{Linear strain} = \frac{1}{m} \times 0.0025$

We also know that increase in thickness (δt),

$$0.04 = t \times \text{Lateral strain} = 50 \times \frac{1}{m} \times 0.0025 = \frac{0.125}{m}$$

$$\frac{1}{m} = \frac{0.04}{0.125} = 0.32 \qquad \textbf{Ans.}$$