Latin Square :

problem 1 :

An agriculturist wants to test the effects of four different fertilizers A, B, C and D on the yield of paddy. In order to eliminate source of error due to variability in itself fertility he used the fertilisers in a latin square arrangements given below. where the numbers indicate yields in quintals per unit area. perform an analysis of variants to decide whether there is a difference between fertilisers at 5% level

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of significance.

A 18	D 21	C 23	B 11
D 22	A 20	B 10	C 19
B 15	C 21	D 25	A 17
C 22	B 12	A 15	D 24

Solution :

$$\text{Origin} = \frac{25+10}{2} = \frac{35}{2} = 17.5 = 18$$

	x_1	x_2	x_3	x_4	Total	x_1^2	x_2^2	x_3^2	x_4^2
y_1	0	3	5	-7	$\Sigma y_1 = 1$	0	9	25	49
y_2	4	2	-8	1	$\Sigma y_2 = -1$	16	4	64	1
y_3	-3	3	7	-1	$\Sigma y_3 = 6$	9	9	49	1
y_4	4	-6	-3	6	$\Sigma y_4 = 1$	16	36	9	36
	$\Sigma x_1 = 5$	$\Sigma x_2 = 2$	$\Sigma x_3 = 1$	$\Sigma x_4 = -1$		$\Sigma x_1^2 = 41$	$\Sigma x_2^2 = 58$	$\Sigma x_3^2 = 147$	$\Sigma x_4^2 = 87$

Step 1 : Formulate H_0 and H_1

H_0 : There is no difference between samples

H_1 : There is difference between samples

Step 2 : Find T and N

$$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 + \Sigma x_4$$

$$= 5 + 2 + 1 - 1$$

$$T = 7$$

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$N = n_1 + n_2 + n_3 + n_4$

$N = 16$

Step 3 : Correction Factor

$$C \cdot F = \frac{T^2}{N} = \frac{(7)^2}{16} = \frac{49}{16}$$

$C \cdot F = 3.06$

Step 4 : Find TSS

$$TSS = \left[\Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2\right] - C \cdot F$$

$$= [41 + 58 + 147 + 87] - 3.06$$

$TSS = 329.94$

Step 5 : SSC

$$SSC = \left[\frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} + \frac{(\Sigma x_4)^2}{n_4}\right] - C \cdot F$$

$$= \left[\frac{(5)^2}{4} + \frac{(2)^2}{4} + \frac{(1)^2}{4} + \frac{(-1)^2}{4}\right] - 3.06$$

$SSC = 4.69$

Step 6 : SSR

$$SSR = \left[\frac{(\Sigma y_1)^2}{n_1} + \frac{(\Sigma y_2)^2}{n_2} + \frac{(\Sigma y_3)^2}{n_3} + \frac{(\Sigma y_4)^2}{n_4}\right] - C \cdot F$$

$$= \left[\frac{(1)^2}{4} + \frac{(-1)^2}{4} + \frac{(6)^2}{4} + \frac{(1)^2}{4}\right] - 3.06$$

$SSR = 6.69$

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Step 7 : SST

					Total
A	0	2	-3	-1	$\Sigma Z_1 = -2$
B	-3	-6	-8	-7	$\Sigma Z_2 = -24$
C	4	3	5	1	$\Sigma Z_3 = 13$
D	4	3	7	6	$\Sigma Z_4 = 20$

$$SST = \left[\frac{(\Sigma Z_1)^2}{t_1} + \frac{(\Sigma Z_2)^2}{t_2} + \frac{(\Sigma Z_3)^2}{t_3} + \frac{(\Sigma Z_4)^2}{t_4}\right] - C.F$$

$$= \left[\frac{(-2)^2}{4} + \frac{(-24)^2}{4} + \frac{(13)^2}{4} + \frac{(20)^2}{4}\right] - 3.06$$

$$SST = 284.19$$

Step 8 : SSE

$$SSE = TSS - [SSC + SSR + SST]$$

$$= 329.94 - [4.69 + 6.69 + 284.19]$$

$$= 329.94 - [11.38 + 284.19]$$

$$= 329.94 - [295.57]$$

$$= 34.37$$

$$\boxed{SSE = 34.37}$$

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Step 9 : ANNOVA Table

Source of variation	Sum of squares	degrees of freedom	Mean sum of squares	Variance ratio
Between columns	$SSC = 4.69$	$c-1$ $4-1$ $= 3$	$MSC = \frac{SSC}{c-1}$ $= \frac{4.69}{3}$ $= 1.563$	$F_C = \frac{5.7291}{1.56}$ $= 3.6666$
Between rows	$SSR = 6.69$	$r-1$ $4-1$ $= 3$	$MSR = \frac{SSR}{r-1}$ $= \frac{6.69}{3} = 2.23$	$F_R = \frac{5.7291}{2.23}$ $= 2.569$
Between Treatments	$SST = 284.19$	$t-1$ $4-1$ $= 3$	$MST = \frac{SST}{t-1}$ $= \frac{284.19}{3} = 94.73$	$F_T = \frac{94.73}{5.7291}$ $= 16.534$
Between errors	$SSE = 34.37$	$(n-1)(n-2)$ $(4-1)(4-2)$ $3 \times 2 = 6$	$MSE = \frac{34.37}{6}$ $= 5.7291$	

Step 10 : Conclusion

$$F_C = 3.666 < 8.94 = F_\alpha(6,3)^{v_1, v_2}$$ $\therefore$ H_0 is accepted

$$F_R = 2.569 < 8.94 = F_\alpha(6,3)$$

$\therefore$ H_0 is accepted

$$F_T = 16.534 > 4.76 = F_\alpha(3,6)$$

$\therefore$ H_0 is rejected

$\therefore$ H_0 is rejected

$\therefore$ There is significance between samples

7. To study the performance of three detergents and 3 different water temperature the following whiteness readings were obtained with designed equipment.

water temperature	detergent A	detergent B	detergent C
Cold water	57	55	67
warm water	49	52	68
Hot water	54	46	58

perform a two way analysis of variance using 5% Level of significance.

Solution:

	x_1	x_2	x_3
A	57	49	54
B	55	52	46
C	67	68	58

Step 1: Formulating H_0 and H_1

H_0: There is no difference between sample

H_1: There is difference between samples

x_1	x_2	x_3	Total	x_1^2	x_2^2	x_3^2
57	49	54	160	3249	2401	2916
55	52	46	153	3025	2704	2116
67	68	58	193	4489	4624	3364
$\Sigma x_1 = 179$	$\Sigma x_2 = 169$	$\Sigma x_3 = 158$		$\Sigma x_1^2 = 10763$	$\Sigma x_2^2 = 9729$	$\Sigma x_3^2 = 8396$

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Step 2: T and N

$$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3$$

$$= 179 + 169 + 158$$

$$T = 506$$

$$N = n_1 + n_2 + n_3$$

$$= 3 + 3 + 3$$

$$N = 9$$

Step 3 : Correction Factor

$$C.F = \frac{T^2}{N} = \frac{(506)^2}{9} = \frac{256036}{9} = 28448.44$$

$$\boxed{C.F = 28448.44}$$

Step 4 : TSS

$$TSS = (\Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2) - C.F$$

$$= (10763 + 9729 + 8396) - 28448.44$$

$$= 28888 - 28448.44$$

$$\boxed{TSS = 439.56}$$

Step 5 : SSC

$$SSC = \left[\frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3}\right] - C.F$$

$$= \left[\frac{(179)^2}{3} + \frac{(169)^2}{3} + \frac{(158)^2}{3}\right] - 28448.44$$

$$= \left[\frac{32041}{3} + \frac{28561}{3} + \frac{24964}{3}\right] - 28448.44$$

$$= [10680.33 + 9520.33 + 8321.33] - 28448.44$$

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$$= 28521.99 - 28448.44$$

$$\boxed{SSC = 73.55}$$

Step 6 : SSR

$$SSR = \left[\frac{(\Sigma y_1)^2}{n_1} + \frac{(\Sigma y_2)^2}{n_2} + \frac{(\Sigma y_3)^2}{n_3} \right] - C.F$$

$$= \left[\frac{(160)^2}{3} + \frac{(153)^2}{3} + \frac{(193)^2}{3} \right] - 28448.44$$

$$= \left[\frac{25600}{3} + \frac{23409}{3} + \frac{37249}{3} \right] - 28448.44$$

$$= (8533.33 + 7803 + 12416.33] - 28448.44$$

$$= 28752.66 - 28448.44$$

$$\boxed{SSR = 304.22}$$

Step 7 : SSE

$$SSE = TSS - [SSC + SSR]$$

$$= 439.56 - [73.55 + 304.22]$$

$$\boxed{SSE = 61.79}$$

Step 8 : ANNOVA Table

Source of Variation	Sum of squares	Degrees of freedom	Mean Square	Variance ratio
Between Columns	$SSC = 73.55$	$c-1$ $3-1$ $=2$	$MSC = \frac{SSC}{(c-1)}$ $= \frac{73.55}{2}$ $= 36.775$	$F_c = \frac{36.775}{15.447}$ $= 2.38$
Between rows	$SSR = 304.22$	$r-1$ $3-1$ $=2$	$MSR = \frac{SSR}{(r-1)}$ $= \frac{304.22}{2}$ $= 152.11$	$F_R = \frac{152.11}{15.447}$ $= 9.847$
within samples	$SSE = 61.79$	$(r-1)(c-1)$ $2 \times 2 = 4$	$MSE = \frac{SSE}{4}$ $= \frac{61.79}{4}$ $= 15.447$	

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Step 9 : conclusion

$$F_c = 2.38 < 19.25 = F_c(2,4)$$

$$F_R = 9.847 < 19.25 = F_R(2,4)$$

$\therefore H_0$ is accepted

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