Randomized Block Design :

problem : 1

Three varieties A, B, C of a crop are tested in a randomized block design. The plot yields in pounds are as follows.

A	6	C	5	A	8	B	9
C	8	A	11	B	6	C	9
B	7	B	6	C	10	A	6

Analyse the experimental yield and state your Conclusion.

Solution : Step 1 : Formulating H_0 and H_1

H_0 : There is no difference between samples.

H_1 : There is difference between samples.

Scanned with CamScanner

x_1	x_2	x_3	x_4	Total	x_1^2	x_2^2	x_3^2	x_4^2
6	11	8	6	31 (Σy_1)	36	121	64	36
7	6	6	9	28 (Σy_2)	49	36	39	81
8	5	10	9	32 (Σy_3)	64	25	100	81
$\Sigma x_1 = 21$	$\Sigma x_2 = 22$	$\Sigma x_3 = 24$	$\Sigma x_4 = 24$	91	$\Sigma x_1^2 = 149$	$\Sigma x_2^2 = 182$	$\Sigma x_3^2 = 200$	$\Sigma x_4^2 = 198$

Step 2 : Find T and N

$$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 + \Sigma x_4$$

$$= 21 + 22 + 24 + 24$$

$$T = 91$$

$$N = n_1 + n_2 + n_3 + n_4$$

$$= 3 + 3 + 3 + 3$$

$$\boxed{N = 12}$$

Step 3 : Correction Factor

$$C.F = \frac{T^2}{N} = \frac{(91)^2}{12} = \frac{8281}{12}$$

$$\boxed{C.F = 690.08}$$

Step 4 : TSS

$$TSS = [\Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2] - C.F$$

$$= [149 + 182 + 200 + 198] - 690.08$$

$$\boxed{TSS = 38.92}$$

Scanned with CamScanner

Step 5 : SSC

$$SSC = \left[\frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} + \frac{(\Sigma x_4)^2}{n_4}\right] - C.F$$

$$= \left[\frac{(21)^2}{3} + \frac{(22)^2}{3} + \frac{(24)^2}{3} + \frac{(24)^2}{3}\right] - 690.08$$

$$\boxed{SSC = 2.22}$$

Step 6 : SSR

$$SSR = \left[\frac{(\Sigma y_1)^2}{n_1} + \frac{(\Sigma y_2)^2}{n_2} + \frac{(\Sigma y_3)^2}{n_3}\right] - C.F$$

$$= \left[\frac{(31)^2}{4} + \frac{(28)^2}{4} + \frac{(32)^2}{4}\right] - 690.08$$

$$\boxed{SSR = 2.17}$$

Step 7: SSE

$$SSE = TSS - [SSC + SSR]$$

$$= 38.92 - [2.22 + 2.17]$$

$$\boxed{SSE = 34.5}$$

Step 8 : ANNOVA Table

Source of variation	Sum of squares	Degrees of freedom	Mean Square	Variance ratio
Between Columns	$SSC = 2.22$	$c-1$ $4-1$ $= 3$	$MSC = \frac{SSC}{c-1}$ $= \frac{2.22}{3}$ $= 0.74$	$F_C = \frac{5.75}{0.74}$ $= 7.77$
Between rows	$SSR = 2.17$	$r-1$ $3-1$ $= 2$	$MSR = SSR/r-1$ $= 2.17/2$ $= 1.085$	$F_R = \frac{5.75}{1.085}$ $= 5.299$
within samples	$SSE = 34.5$	$(r-1)(c-1)$ $2 \times 3 = 6$	$MSE = \frac{SSE}{(r-1)(c-1)}$ $= 5.75$	

Scanned with CamScanner

Step 9 : Conclusion

$F_C = 7.77 > 4.76 = F_C(3,6)$

$F_R = 5.299 > 5.14 = F_R(2,6)$

$\therefore H_0$ is rejected

Problem : 2

A certain company had four salesmen A, B, C and D, each of whom was sent for a week into three types of area X, Y and Z. Their sales (in Rs) per week are shown in the following table.

Area	Salesmen			
	A	B	C	D
X	30	70	30	30
Y	80	50	40	70
Z	100	60	80	80

Test whether there is any significant difference between salesmen. Also test whether there is any difference between areas.

Solution :

x_1	x_2	x_3	x_4	Total	x_1^2	x_2^2	x_3^2	x_4^2
30	70	30	30	160 (Σy_1)	900	4900	900	900
80	50	40	70	240 (Σy_2)	6400	2500	1600	4900
100	60	80	80	320 (Σy_3)	10,000	3600	6400	6400
$\Sigma x_1 = 210$	$\Sigma x_2 = 180$	$\Sigma x_3 = 150$	$\Sigma x_4 = 180$		$\Sigma x_1^2 = 17300$	$\Sigma x_2^2 = 11000$	$\Sigma x_3^2 = 8900$	$\Sigma x_4^2 = 12200$

Scanned with CamScanner

Step 1 : Formulate H_0 and H_1

H_0 : There is no difference between samples.

H_1 : There is difference between samples

Step 2 : Find T and N

$$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 + \Sigma x_4$$

$$= 210 + 180 + 150 + 180$$

$$\boxed{T = 720}$$

$$N = n_1 + n_2 + n_3 + n_4$$

$$= 3 + 3 + 3 + 3$$

$$\boxed{N = 12}$$

Step 3 : Find correction Factor

$$C.F = \frac{T^2}{N} = \frac{(720)^2}{12} = \frac{518400}{12} = 43,200$$

$$\boxed{C.F = 43,200}$$

Step 4 : Find TSS

$$TSS = [\Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2] - C.F$$

$$= (17300 + 11000 + 8900 + 12200] - 43200$$

$$= 49400 - 43200$$

$$\boxed{TSS = 6200}$$

Step 5 : Find SSC

$$SSC = \left[\frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} + \frac{(\Sigma x_4)^2}{n_4}\right] - C.F$$

$$= \left[\frac{(210)^2}{3} + \frac{(180)^2}{3} + \frac{(150)^2}{3} + \frac{(180)^2}{3}\right] - 43200$$

Scanned with CamScanner

$$= \left[\frac{44100}{3} + \frac{32400}{3} + \frac{22500}{3} + \frac{32400}{3}\right] - 43200$$

$$= [14700 + 10800 + 7500 + 10800] - 43200$$

$$= 43800 - 43200$$

$$\boxed{SSC = 600}$$

Step 6 : Find SSR

$$SSR = \left[\frac{(\Sigma y_1)^2}{n_1} + \frac{(\Sigma y_2)^2}{n_2} + \frac{(\Sigma y_3)^2}{n_3}\right] - C.F$$

$$= \left[\frac{(160)^2}{4} + \frac{(240)^2}{4} + \frac{(320)^2}{4}\right] - 43200$$

$$= \left[\frac{25600}{4} + \frac{57600}{4} + \frac{102400}{4}\right] - 43200$$

$$= [6400 + 14400 + 25600] - 43200$$

$$= 46400 - 43200$$

$$\boxed{SSR = 3200}$$

Step 7 : Find SSE

$$SSE = TSS - [SSC + SSR]$$

$$= 6200 - [600 + 3200]$$

$$= 6200 - 3800$$

$$\boxed{SSE = 2400}$$

Scanned with CamScanner

Step 8 : ANNOVA Table

Source of variation	Sum of Squares	degrees of freedom	Mean Square	variance ratio
Between columns	$SSC = 600$	$C-1$ $4-1$ $= 3$	$MSC = \frac{SSC}{C-1}$ $= 600/3$ $= 200$	$F_C = \frac{400}{200}$ $= 2$
Between rows	$SSR = 3200$	$r-1$ $3-1$ $= 2$	$MSR = \frac{SSR}{r-1}$ $= 3200/2$ $= 1600$	$F_R = \frac{1600}{400}$ $= 4$
within samples	$SSE = 2400$	$(r-1)(c-1)$ 2×3 $= 6$	$MSE = \frac{SSE}{(r-1)(c-1)}$ $= 400$	

Step 9 : conclusion

$$F_C = 2 < 4.76 = F_C\overset{v_1,\, v_2}{(3, 6)}$$

$$F_R = 4 < 5.14 = F_R(2, 6)$$

$\therefore$ H_0 is accepted