Newton's integration by trapezoidal, Simpso

$\frac{1}{3}$ rule and $\frac{3}{8}$ rule:

Trapezoidal rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\{(y_0+y_n)+2(y_1+y_2+\cdots y_{n-1})\}.$$

$$= \frac{h}{2}(A+2B)$$

Where, A ⇒ 1st and sum of the 1st and last ordinates

B ⇒ sum of the remaining ordinates

Simpson $\frac{1}{3}$ rule : (Even intervals).

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\{(y_0+y_n)+2(y_2+y_4+y_6+\cdots y_{n-2})+4(y_1+y_3+\cdots y_{n-1})\}$$

$$= \frac{h}{3}\{A+2B+4C\}.$$

where $A \Rightarrow$ Sum of 1st and last co-ordinate

$B \Rightarrow$ Sum of even ordinates

$C \Rightarrow$ Sum of odd ordinates

[An equal intervals of even sub intervals].

(65) Using trapezoidal rule evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

$$h = \frac{x_n - x_0}{n} = \frac{1+1}{8} = \frac{2}{8} = \frac{1}{4} = 0.25$$

x	-1	-0.75	-0.5	-0.25	0	0.25	0.5
y	0.5	0.64	0.8	0.941	1	0.941	0.8

0.75	1
0.64	0.5

By trapezoidal rule,

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\{(y_0 + y_8) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)$$

$$\int_{-1}^{1} \frac{dx}{1+x^2} = \frac{0.25}{2}\{(0.5+0.5)+2(0.64+0.8+0.941+1+0.941+0.8+0.64)\}$$

$$= 1.5655$$

66) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ with $h=\frac{1}{6}$ by trapezoidal rule.

$h = \frac{x_n - x_0}{n} \Rightarrow \frac{1}{6} = \frac{1}{n}$ $n=6$.

x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
y	1	0.973	0.9	0.8	0.692	0.59	0.5

By trapezoidal rule,

$$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{12}\{(1+0.5)+2(0.973+0.9+0.8+0.692+0.59)\}$$

$$= 0.7842.$$

67) Evaluate the interval $\int_0^2 \frac{dx}{1+x^2}$ using the trapezoidal rule with 5 ordinate $n=2$.

68) Dividing the range into 10 equal parts find the value $\int_0^{\pi/2} \sin x\, dx$ by (i) trapezoidal rule (ii) Simpsons $\frac{1}{3}$ rule

$n=10$, $y=\sin x$ $x_0=0$ $x_n=\frac{\pi}{2}$ $h=\frac{\pi}{20}$

x	0	$\frac{\pi}{20}$	$\frac{2\pi}{20}$	$\frac{3\pi}{20}$	$\frac{4\pi}{20}$	$\frac{5\pi}{20}$	$\frac{6\pi}{20}$	$\frac{7\pi}{20}$	$\frac{8\pi}{20}$	$\frac{9\pi}{20}$	$\frac{\pi}{2}$
y	0	0.156	0.309	0.454	0.588	0.707	0.809	0.891	0.951	0.988	1

(i) $$\int_0^{\pi/2} \sin x\, dx = \frac{0.157}{2}\{(0+1)+2(0.156+0.309+0.454+0.588+0.707+0.809+0.951+0.988)\}$$

$\int_0^{\pi} \sin x\,dx = 0.9974.$

(ii) $\int_0^{\pi/2} \sin x\,dx = \frac{0.157}{3}\{(0+1)+2(0.309+0.588+0.809+0.951)+4(0.156+0.454+0.707+0.891+0.988)\}.$

$= 0.9995.$

Find the value of $\log_e 5$ from $\int_0^5 \frac{dx}{4x+5}$ by Simpson's $\frac{1}{3}$ rule dividing the range into 10 equal parts. $x_0 = 0$ $x_n = 5$. $n = 10$

$h = \frac{x_n - x_0}{n} = \frac{5-0}{10} = 0.5$. Here, $y = \frac{1}{4x+5}$

	0	1	2	3	4	5	6
x	0	0.5	1	1.5	2	2.5	3
y	0.2	0.143	0.111	0.09	0.077	0.067	0.059

7	8	9	10
3.5	4	4.5	5
0.053	0.048	0.043	0.04

By Simpson's $\frac{1}{3}$ rule,

$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\{(y_0+y_n)+2(y_2+y_4+y_6+y_8)+4(y_1+y_3+y_5+y_7+y_9)\}$

$\int_0^5 \frac{dx}{4x+5} = \frac{0.5}{3}\{(0.2+0.04)+2(0.111+0.077+0.059+0.048)+4(0.143+0.09+0.067+0.053+0.043)\}$

$\int_0^5 \frac{dx}{4x+5} = 0.402$

$\left[\frac{\log_e(4x+5)}{\quad}\right]_0^5 = 0.402.$

$$\frac{\log_e 25 - \log_e 5}{4} = 0.402$$

$$\log_e 25 - \log_e 5 = 1.608$$

$$\log_e 5 = \log_e 25 - 1.608.$$

$$\boxed{\log_e 5 = 1.61}$$

7) $\int_1^2 \frac{dx}{1+x^2}$ $n=2$.

Here, $y = \frac{1}{1+x^2}$

$x_0 = 1$ $x_n = 2$

$h = \frac{x_n - x_0}{n} = \frac{2-1}{2} = 0.5$

x	1	1.5	2
y	0.5	0.308	0.2

By trapezoidal rule.

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\{(y_0+y_2) + 2(y_1)\}$$

$$\int_1^2 \frac{1\,dx}{1+x^2} = \frac{0.5}{2}\{(0.5+0.2) + 2(0.308)\}$$

$$\int_1^2 \frac{dx}{1+x^2} = 0.329.$$

12) Using simpsons $\frac{1}{3}$ rule evaluate $\int_0^1 xe^x dx$ taking 4 intervals. compare with actual value.

$x_0 = 0$ $x_n = 1$. $n = 4$.

$h = \frac{x_n - x_0}{n} = \frac{1-0}{4} = 0.25$

	0	1	2	3	4
x	0	0.25	0.5	0.75	1
y	0	0.321	0.824	1.588	2.718

By simpson's $\frac{1}{3}$ rule,

$$\int_0^1 xe^x dx = \frac{0.25}{3}[0+2.718+2(0.824)+4(0.321+1.588)]$$

$$= 1.0001$$

Actual value = 1

Calculate $\int_{0.5}^{0.7} e^{-x}\sqrt{x}\,dx$ taking 5 ordinates by simpson $\frac{1}{3}$ rule.

$x_0 = 0.5 \quad x_n = 0.7 \quad n = 4.$

$$h = \frac{0.7-0.5}{4} = 0.05$$

	0	1	2	3	4
x	0.5	0.55	0.6	0.65	0.7
y	0.429	0.428	0.425	0.421	0.415

By simpsons $\frac{1}{3}$ rule,

$$\int_{0.5}^{0.7} e^{-x}\sqrt{x}\,dx = \frac{0.05}{3}[(0.429+0.415)+2(0.425)+4(0.428+0.421)]$$

$$= 0.085$$

By dividing the range into 10 equal parts evaluate $\int_0^{\pi} \sin x\,dx$ by trapezoidal rule and simpson rule. compare the value with actual value.

$x_0 = 0 \quad x_n = \pi \quad n = 10$

$h = \frac{\pi - 0}{10} = \frac{\pi}{10} = 18^\circ = 0.314$ rad.

x	0	18°	36°	54°	72°	90°
y	0	0.309	0.588	0.809	0.951	1

108°	126°	144°	162°	180°
0.951	0.809	0.588	0.309	0

By trapezoidal rule.

$$\int_0^{\pi} \sin x\,dx = \frac{0.314}{2}[2(0.309+0.588+0.809+0.951+1+0.951+0.809+0.588+0.309)]$$

$= 1.983.$

By Simpson's $\frac{1}{3}$ rule

$$\int_0^{\pi} \sin x\,dx = \frac{0.314}{3}[2(0.588+0.951+0.951+0.588)+4(0.309+0.809+1+0.809+0.309)]$$

$= 2.00014$