Solution of Linear System by Gauss elimination & Gauss Jordon method...

Prob.N: There are two types of methods to Solve Simultaneous linear Algebric equation with many unknowns.

2 Methods.

Direct Method.

Indirect Method.
(Iterative Method).

Direct Method:

1). Gauss elimination
2). Gauss Jordon.

Indirect Method:

1). Gauss Jacobi
2). Gauss Seidal.

*. Gauss elimination Method is a direct Method which consists of transforming the given System of Simultaneous equation to a upper triangular System. From this System the required Solution can be Obtained by the method of back Substitution.

*. Gauss Jordon Method - The Solution of the System of equation by reducing the Matrix to a diagonal Matrix is known as Gauss Jordon Method.

(Direct Method). (to calculat x, y, z value)

Prob.1: Solve the System of equation by Gauss elimination method & Gauss jordon Method.

1). $10x-2y+3z=23$ 2). $2x+10y-5z=-33$

3). $3x-4y+10z=41$.

(i). GAUSS ELIMINATION METHOD:

$\boxed{Ax=B}$

$$\begin{pmatrix} 10 & -2 & 3 \\ 2 & 10 & -5 \\ 3 & -4 & 10 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 23 \\ -33 \\ 41 \end{pmatrix}.$$

By Augmented Matrix;

[Now, Fix 1st row & change the 2nd & 3rd row.

$$\Rightarrow \left(\begin{array}{ccc|c} 10 & -2 & 3 & 23 \\ 0 & 52/5 & -28/5 & -188/5 \\ 0 & -34/10 & 91/10 & -28/10 \end{array}\right) \begin{array}{l} R_2 \to R_2 - 2/10(R_1) \\ R_3 \to R_3 - 3/10(R_1) \end{array}$$]

(or)

By Ranking method:

Fix, 1st row & change 2nd & 3rd row.

$$\Rightarrow \left(\begin{array}{ccc|c} 10 & -2 & 3 & 23 \\ 0 & 52 & -28 & -188 \\ 0 & -34 & 91 & 341 \end{array}\right) \begin{array}{l} R_2 \to 5R_2 - R_1 \\ R_3 \to 10R_3 - 3R_1 \end{array}$$

Now, Fix, 2nd row; change 3rd row.

$$\Rightarrow \left(\begin{array}{ccc|c} 10 & -2 & 3 & 23 \\ 0 & 52 & -28 & -188 \\ 0 & 0 & 3780 & 11340 \end{array}\right) \rightarrow (1) \quad R_3 \to 52R_3 + 34R_2$$

upper-triangular matrix

By Back Substitution Method:

* $3780z = 11340$

$\Rightarrow \boxed{z = 3}$

* $52y - 28z = -188$

$52y - 28(3) = -188$

$52y - 84 = -188$

$\Rightarrow 52y = -188 + 84$

$y = \dfrac{-188+84}{52}$

$\Rightarrow y = \dfrac{-104}{52}$

$\boxed{y = -2}$

* $10x - 2y + 3z = 23$

$10x - 2(-2) + 3(3) = 23$

$10x + 4 + 9 = 23$

$10x = 23 - 13 \Rightarrow x = 10/10$

$\therefore (x, y, z) = (1, -2, 3) \Rightarrow (2) \quad \Rightarrow \boxed{x = 1}$

(ii) GAUSS JORDHON METHOD: (Convert the eq(1) to diagonal matrix).

$$(1) \Rightarrow \left(\begin{array}{ccc|c} 10 & -2 & 3 & 23 \\ 0 & 52 & -28 & -188 \\ 0 & 0 & 3780 & 11340 \end{array}\right)$$

From (1), Fix 3rd row & change 1st and 2nd row.

$$\left(\begin{array}{ccc|c} 10 & -2 & +3 & 23 \\ 0 & 13 & -7 & -47 \\ 0 & 0 & 3780 & 11340 \end{array}\right) \begin{array}{l} R_2 \to R_2/4 \\ R_3 \to R_3/3780 \end{array}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 10 & -2 & +3 & 23 \\ 0 & 13 & -7 & -47 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 10 & -2 & 0 & 14 \\ 0 & 13 & 0 & -26 \\ 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{l} R_1 \to R_1 - 3R_3 \\ R_2 \to R_2 + 7R_3 \end{array}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 130 & 0 & 0 & 130 \\ 0 & 13 & 0 & -26 \\ 0 & 0 & 1 & 3 \end{array}\right) R_1 \to 13R_1 + 2R_2.$$

(diagonal matrix)

$130x = 130$ | $13y = -26$ | $1z = 3$

$\boxed{x = 1}$ | $\boxed{y = -2}$ | $\boxed{z = 3}$

$\therefore (x, y, z) = (1, -2, 3) \to (3)$.

From equation (2) & (3), we get $(x,y,z) = (1,-2,3)$

$\therefore$ Hence it proves.

Gauss elimination = Gauss Jordhon method.

Problem.2.. Solve the system of equation by Gauss elimination and Gauss Jordhon method...

i). $10x + y + z = 12$.

ii). $2x + 10y + z = 13$.

iii). $x + y + 5z = 7$.

Consider $\boxed{Ax=B}$.

$$\Rightarrow \begin{pmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 13 \\ 7 \end{pmatrix}$$

By Augmented Method:..

$$[A/B] \Rightarrow \left(\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{array}\right)$$

By Ranking Method:.

$\therefore$ Fix, 1^{st} row & change 2^{nd} & 3^{rd} row.

$$\left(\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 9 & 49 & 58 \end{array}\right) \begin{array}{l} R_2 \Rightarrow 5R_2 - R_1 \\ R_3 \Rightarrow 10R_3 - R_1 \end{array}$$

$\therefore$ Now, Fix 2^{nd} row & change 3^{rd} row.

To ~~Find~~ get diagonal Matrix :.

$$[A/B] = \left(\begin{array}{ccc|c} 490 & 0 & 45 & 535 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 2365 & 2365 \end{array}\right) \begin{array}{l} R_1 \rightarrow 49R_1 - R_2 \\ R_3 \rightarrow 49R_3 - 9R_2 \end{array}$$

$$[A/B] = \left[\begin{array}{ccc|c} 98 & 0 & 9 & 107 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1/5 \\ \\ R_3/2365 \end{array}$$

$\therefore$ Fix the 3^{rd} row, and change 1^{st} & 2^{nd} row.

$$[A/B] = \begin{pmatrix} 98 & 0 & 0 \\ 0 & 49 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 98 \\ 49 \\ 1 \end{pmatrix} \begin{array}{l} R_1 \rightarrow R_1 - 9R_3 \\ R_2 \rightarrow R_2 - 4R_3 \end{array}$$

$98x = 98$	$49y = 49$	$1z = 1$
$\boxed{x=1}$	$\boxed{y=1}$	$\boxed{z=1}$

$\therefore (x, y, z) = (1, 1, 1)$

Hence, it is proved by Satisfying Gauss elimination method and Gauss Jordon method...

Problem 3 : Solve the system of equation by Gauss Jordan method.

i). $x+3y+3z=16$

ii) $x+4y+3z=18$

iii). $x+3y+4z=19$.

Consider $\boxed{AX=B}$

$$\Rightarrow \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16 \\ 18 \\ 19 \end{pmatrix}.$$

By Augmented Method :.

$$(A/B) = \left(\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 1 & 4 & 3 & 18 \\ 1 & 3 & 4 & 19 \end{array}\right).$$

By Ranking Method :.

∴ Fix 1^{st} row & change 2^{nd} & 3^{rd} row.

$$(A/B) = \left(\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

To Find diagonal Matrix :.

∴ Fix R_2 & change R_1

$$(A/B) = \left(\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right) R_1 \to R_1 - 3R_2$$

∴ Fix R_3 & change R_1 :

$$(A/B) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} R_1 \to R_1 - 3R_3$$

$\boxed{x=1}$ $\boxed{y=2}$ $\boxed{z=3}$.

$(x, y, z) = (1, 2, 3)$.

∴ Hence proved...

Problem. 4 : (2M) Solve the System of equation by Gauss Jordon method

i) $5x+4y=15$

ii). $3x+7y=12$.

$AX=B. \Rightarrow \begin{pmatrix} 5 & 4 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 15 \\ 12 \end{pmatrix}$

Augmented Matrix :

$(A/B) = \left(\begin{array}{cc|c} 5 & 4 & 15 \\ 3 & 7 & 12 \end{array}\right)$ $R_1 \to 7R_1 - 4R_2$, $R_2 \to 3R_1 - 5R_2$.

$\Rightarrow \left(\begin{array}{cc|c} 23 & 0 & 57 \\ 0 & -23 & -15 \end{array}\right)$.

$\Rightarrow \begin{pmatrix} 23 & 0 \\ 0 & -23 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 57 \\ -15 \end{pmatrix}$.

$\Rightarrow 23x = 57$ | $\Rightarrow -23y = -15$.

$x = 57/23$ | $\therefore y = 15/23$

$\Rightarrow \boxed{x = 2.47}$ $\boxed{y = 0.65}$