Test of significance of small sample.

Student's t - test

Test for single mean :

Null hypothesis : $H_0 = \mu = \mu_0$

Test Statistic : $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$ if SD is given

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

To find $s = \dfrac{\Sigma (x - \bar{x})^2}{n-1}$

Degree of freedom : $v = n-1$

Confidence limit : $\bar{x} \mp t_\alpha \dfrac{s}{\sqrt{n-1}}$

Example ①

A random sample of 10 boys has the following IQs 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of population mean IQs of 100. ~~Do~~ ~~these~~ Find reservable range in which most of the mean IQs value of sample 10 boys.

$n = 10$, $\mu = 100$

$$\bar{x} = \frac{70+120+110+101+88+83+95+98+107+100}{10}$$

$$\bar{x} = \frac{972}{10} = 97.2$$

x :	70	120	110	101	88	83	95	98	107	100
$(x-\bar{x})$:	-27.2	22.8	12.8	3.8	-9.2	-14.2	-2.2	0.8	9.8	2.8
$(x-\bar{x})^2$:	739.84	519.84	163.84	14.44	84.64	201.64	4.84	0.64	96.04	7.84

$$s^2 = \frac{\sum(x-\bar{x})^2}{n-1} = \frac{1833.6}{10-1} \Rightarrow 203.73$$

$$\boxed{S = 14.27}$$

Step 1: Formulate H_0, H_1

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$ [two tail]

Step 2: LOS = 5% → 0.05

Step 3: Test statistic

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \Rightarrow \frac{97.2 - 100}{\frac{14.27}{\sqrt{10}}} = -0.62$$

$|t| = |-0.62| = 0.62$

Step 4: t_{tab} for degree of freedom.

$$V = n-1$$
$$V = 10-1$$
$$V = 9$$

$$t_{tab} = 2.262 = t_\alpha$$

Step 5: conclusion

$$|t| = 0.62 < 2.262 = t_\alpha$$

$\therefore$ H_0 is accepted.

Confidente limit :

$$\bar{x} \pm t_\alpha \cdot \frac{s}{\sqrt{n-1}}$$

$$= 97.2 \pm 2.262 \times \frac{14.27}{\sqrt{10-1}}$$

$$= 97.2 \pm 10.759$$

$$= 107.95,\ 86.45$$

2) The weight of 10 peoples of a locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 kg it is reasonable to believe that the average weight of people locality greater than 64 kg. test at 5% LOS.

Soln:

Given : $n = 10$, $\mu = 64$

$$\bar{x} = \frac{70+67+62+68+61+68+70+64+64+66}{10}$$

$$\bar{x} = 66$$

To find s

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$$

x :	70	67	62	68	61	68	70	64	64	66
$x-\bar{x}$:	4	1	-4	2	-5	2	4	-2	-2	0
$(x-\bar{x})^2$	16	1	16	4	25	4	16	4	4	0

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{90}{10-1} = 10$$

$s = 3.16$

Step 1: Formulating H_0 and H_1:

$H_0 : \mu = 64$

$H_1 : \mu > 64$ (one tail test - right)

Step 2: LOS at α = 5%

Step 3: Test statistic, $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{66 - 64}{3.16/\sqrt{10}}$

$= 2.02$

Step 4: t_{tab} for degree of freedom, $\nu = n-1$

$= 10-1$

$= 9$

(ie) t_{tab} : $t_\alpha = 1.833$ (at two tailed at 10%)

Step 5: Conclusion : $t = 2.02 > 1.833 = t_\alpha$

$\therefore H_0$ is rejected at 5% LOS.

(ie) the avg. weight of people locality is greater than 64. kg.

TEST FOR DIFFERENCE OF MEAN :

Null hypothesis : $H_0 : \mu_1 = \mu_2$

Test statistic $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Where $s^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$ (or) $s^2 = \dfrac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$

Degrees of freedom : $\nu = n_1 + n_2 - 2$

$s^2 = \dfrac{\Sigma(x - \bar{x}_1)^2 + \Sigma(x - \bar{x}_2)^2}{n_1 + n_2}$

Example (1)

In a test examination given ~~the~~ two groups of students. The marks obtained where as follows

Group 1 : 18 20 36 50 49 36 34 49 41

Group 2 : 29 28 26 35 30 44 46

Examine whether the significance of difference between the average marks secured by the students of the above two groups.

Given: $n_1 = 9$; $n_2 = 7$

$$\bar{x}_1 = \frac{18+20+36+50+49+36+34+49+41}{9} = \frac{333}{9} = 37$$

$$\bar{x}_2 = \frac{29+28+26+35+30+44+46}{7} = \frac{238}{7} = 34$$

~~Std~~ $\cancel{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}}$ $\quad s^2 = \frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}$

x_1	$x_1-\bar{x}_1$	$(x_1-\bar{x}_1)^2$	x_2	$x_2-\bar{x}_2$	$(x_2-\bar{x}_2)^2$
18	-19	361	29	-5	25
20	-17	289	28	-6	36
36	-1	1	26	-8	64
50	13	169	35	1	1
49	12	144	30	-4	16
36	-1	1	44	10	100
34	-3	9	46	12	144
49	12	144			
41	4	16			
		1134			386

$$s^2 = \frac{1134 + 386}{9+7} = 108.5714$$

$$s = 10.4197 \simeq 10.42$$

Step 1: Formulate H_0 & H_1

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (Two tailed)

Step 2: LOS 5% = 0.05

Step 3: Test statistics

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{37 - 34}{10.\cancel{3}42\sqrt{\frac{1}{9} + \frac{1}{7}}}$$

$$t = 0.571$$

Step 4: Degree of freedom : $V = n_1 + n_2 - 2$

$= 9 + 7 - 2$

$= 14$

$$t_{tab} = 3.145$$

Step 5: Conclusion:

$$t = 0.571 < 2.145 = t_{tab}$$

$\therefore$ Ho is accepted.

2) A samples of two types of electric bulbs were tested for length of life and the following data were obtained.

Samples	size	Mean	SD
I	8	1134	35
II	7	1024	40

Test at 5%.

Scanned with CamScanner

Given:

Sample I : $n_1 = 8$, $\bar{x}_1 = 1134$, $s_1 = 35$

Sample II : $n_2 = 7$, $\bar{x}_2 = 1024$, $s_2 = 40$

Step I : Formulating H_0 and H_1

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (Two tailed test)

Step 2 : LOS at $\alpha = 5\% = 0.05$

Step 3 : Test slastics

$$\therefore t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Now $$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{8(35)^2 + 7(40)^2}{8+7-2}$$

$$= 1615.38$$

$$S = 40.19$$

$$\therefore t = \frac{1134 - 1024}{40.19\sqrt{\frac{1}{8} + \frac{1}{7}}}$$

$$\Rightarrow \frac{110}{20.8} = 5.288$$

Step 4 : t_{tab} for degree of freedom, $V = n_1 + n_2 - 2$

$= 8 + 7 - 2$

$= 13$

(i) t_{tab} : $t_\alpha = 2.160$

Step 5 : Conclusion : $t = 5.288 > 2.160 = t_\alpha$

$\therefore$ H_0 is rejected at 5%