Test For Difference Of Proportions:

Null hypothesis $H_0 : P_1 = P_2$

Test Statistic $Z = \dfrac{P_1' - P_2'}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

Where $P_1' = \dfrac{x_1}{n_1}$; $P_2' = \dfrac{x_2}{n_2}$

and $p = \dfrac{P_1' n_1 + P_2' n_2}{n_1 + n_2} = \dfrac{x_1 + x_2}{n_1 + n_2}$ & $q = 1 - p$

Example (1):

Random samples of 400 men and 600 women where asked whether they whould have a ~~fl~~avour near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportion of men and women in favour of the proposal are same against that they are not at 5% level.

$n_1 = 400$ $\quad x_1 = 200$

$n_2 = 600$ $\quad x_2 = 325$

$P_1' = \dfrac{x_1}{n_1} = \dfrac{200}{400} = \dfrac{1}{2} = 0.5$

$P_2' = \dfrac{x_2}{n_2} = \dfrac{225}{600} = 0.541$

$P = \dfrac{x_1 + x_2}{n_1 + n_2} \Rightarrow \dfrac{525}{1000} = 0.525$

$q = 1 - P \Rightarrow q = 1 - 0.525 = 0.425$

Step 1 : Formulate H_0 & H_1

$H_0 : P_1 = P_2$

$H_1 : P_1 \neq P_2$ (Two tailed)

Step 2 : $LOS_{\alpha} = 0.05$

Step 3 : Test the statistic

$$Z = \frac{P_1' - P_2'}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \Rightarrow \frac{0.5 - 0.541}{\sqrt{(0.525)(0.425)\left(\frac{1}{400}+\frac{1}{600}\right)}}$$

$$= \frac{-0.041}{\sqrt{0.001039}} \Rightarrow -1.269$$

$|Z| = 1.269$

Step 4 : Critical Value

$Z_{\alpha} = 1.96$

Step 5 : Conclusion.

$Z = 1.269 < 1.96 = Z_{\alpha}$

$\therefore$ H_0 is accepted.

Example (2) :

In a large city A, 20% random sample of 900 school children had defective eye site. In other large city B 15% of random sample of 1600 children had the same defect. In this difference between proposition significant? (Obtain 95% confidence limits for the difference in the population proposition.

$n_1 = 900$ $\quad\quad$ $p_1' = 20\% = 0.20$ $\quad\quad$ $\cancel{q = 1 - p =}$

$n_2 = 1600$ $\quad\quad$ $p_2' = 15\% = 0.15$

$$\cancel{p = \frac{p_1' - p_2'}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.20 - 0.15}{\sqrt{\quad}}}$$

$$P = \frac{(0.20)(900) + (0.15)(1600)}{900 + 1600} = \frac{180 + 240}{2500} = 0.168$$

$P = 0.168$ $\quad\quad$ $q = 1 - 0.168 = 0.832$

Step 1 : Formulate H_0 & H_1

$H_0 : P_1 = P_2$

$H_1 : P_1 \neq P_2$ (Two tailed test)

Step 2: $LOS_\alpha = 0.05$

Step 3: Test of ~~stastics~~. statistic

$$Z = \frac{P_1' - P_2'}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.20 - 0.15}{\sqrt{(0.168)(0.832)\left(\frac{1}{900} + \frac{1}{1600}\right)}}$$

$$= \frac{0.05}{\sqrt{0.139776\left(\frac{1}{900} + \frac{1}{1600}\right)}}$$

$Z = 3.21$

Step 4 : Critical value

$Z_\alpha = 1.96$

Step 5 : Conclusion.

$Z = 3.21 > 1.96 = Z_\alpha$

$\therefore$ H_0 is rejected.

H_1 is accepted.