

Test for single proportion Mean:

If x is the number of persons (items) a certain attributes in a sample of 'n' items (or) persons then the sample proportion $\hat{p} = \frac{x}{n}$

Null hypothesis $H_0: P = P_0$, where P is population proportion

$$H_1: P \neq P_0$$

Test statistics $Z: \frac{\hat{p} - P_0}{\sqrt{\frac{P_0 q_0}{n}}}$, where $q = 1 - P$.

Example 1.

A coin is tossed 256 times and 132 heads are obtained. Would you conclude that the coin is a biased one.

$$n = 256$$

$$x = 132$$

$$p' = \frac{x}{n} = \frac{132}{256} = 0.5156$$

$$P = \frac{1}{2}, \quad q = 1 - P$$

$$q = \frac{1}{2}$$

Step 1: Formulate the hypothesis H_0 & H_1

$$H_0 : P = \frac{1}{2}$$

$$H_1 : P \neq \frac{1}{2} \text{ (Two tail)}$$

Step 2: Level of significance (α) = 5% = 0.05

Step 3: Test statistic

$$Z = \frac{p' - P}{\sqrt{\frac{Pq}{n}}}$$

$$= \frac{0.5156 - 0.5}{\sqrt{\frac{0.25}{256}}}$$

$$Z = 0.4992$$

$$|Z| = 0.4992$$

Step 4: Critical value

$$Z_{\alpha} = 1.96$$

Step 5: Conclusion.

$$|Z| = 0.4992 < 1.96 = Z_{\alpha}$$

$\therefore H_0$ is accepted.

H_1 is rejected.

Example 2:

20 people were affected by cholera and out of them only 18 people were survive. Would you reject the hypothesis that the survival rate, if affected by cholera is 85% in favour of the hypothesis that it is more than 5% level of significance

$$n = 20$$

$$x = 18$$

$$p' = \frac{x}{n} = \frac{18}{20} = 0.9$$

$$P = \frac{85}{100} = 0.85$$

$$q = 1 - P$$

$$q = 1 - 0.85$$

$$q = 0.15$$

Step 1: Formulate the hypothesis H_0 & H_1

$$H_0: P = 0.85$$

$$H_1: P > 0.85 \text{ (Right one tail)}$$

Step 2: $LOS_{\alpha} = 5\% = 0.05$

Step 3: Test statistic

$$Z = \frac{p' - p}{\sqrt{\frac{pq}{n}}}$$

$$Z = \frac{0.9 - 0.85}{\sqrt{\frac{0.85 \times 0.15}{20}}}$$

$$= \frac{0.05}{\sqrt{\frac{0.1275}{20}}} = \frac{0.05}{\sqrt{6.375}} = \frac{0.05}{2.5248}$$

$$= \frac{0.05}{0.0798}$$

$$Z = 0.6265$$

Step 4: Critical value $Z_{\alpha} = 1.645$

Step 5: Conclusion $|z| = 0.6265 < 1.645 = Z_{\alpha}$

H_0 is accepted.

H_1 is rejected.

Example 3:

In sample of 500 people in Kerala 280 are tea drinker and rest of the people are coffee drinker. It can be assumed that both tea and coffee are popular in the state. (LOS is 5%)

$$N = 500$$

$$n = 280$$

$$p' = \frac{x}{n} = \frac{280}{500} = 0.56$$

$$P = \frac{1}{2} = 0.5$$

$$q = 1 - P = 1 - 0.5 = 0.5$$

Step 1: Formulate H_0 & H_1

$$H_0: P = \frac{1}{2}$$

$$H_1: P \neq \frac{1}{2} \text{ [Two tailed]}$$

Step 2:

$$\text{LOS}_{\alpha} = 5\% = 0.05$$

Step 3: Test of statistic

$$Z = \frac{p' - P}{\sqrt{\frac{Pq}{n}}} = \frac{0.56 - 0.5}{\sqrt{\frac{0.25}{500}}} = \frac{0.06}{\sqrt{0.0005}}$$

$$Z = \frac{0.06}{0.022} = 2.72$$

$$|Z| = 2.722$$

Step 4: Critical value $Z_{\alpha} = 1.96$

Step 5: Conclusion

$$|Z| = 2.727$$

$$Z_{\alpha} = 1.96$$

$$|Z| > Z_{\alpha}$$

$\therefore H_0$ is rejected.