



DEPARTMENT OF MATHEMATICS

Function of random variable [Transformation of one dimensional random variable]

$$f(y) = \left| \frac{dx}{dy} \right| f(x)$$

or

$$f_x(y) = \left| \frac{dx}{dy} \right| f_x(x)$$

7th ques

1) Let x be a continuous random variable with pdf

$$f(x) = \begin{cases} \frac{1}{2} & , 1 < x < 5 \\ 0 & , \text{otherwise} \end{cases} \quad \text{find pdf of } y = 2x - 3$$

$$f(y) = \left| \frac{dx}{dy} \right| f(x)$$

$$y = 2x - 3$$

$$2x = y + 3$$

$$x = \frac{y+3}{2}$$

$$\frac{dx}{dy} = \frac{1}{2} (1)$$

$$f(y) = \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{1}{4} \left(\frac{y+3}{2} \right)$$

$$= \frac{1}{8} (y+3)$$

$$f(y) = \frac{y+3}{8} \quad , \quad -1 < y < 7$$

limit: $x = 1$ $x = 5$

$$y = 2 - 3$$

$$y = 10 - 3$$

$$y = -1$$

$$y = 7$$



2) If the pdf of x is $f(x) = e^{-x} - x > 0$. Find the pdf of $y = 2x + 1$.

$$f(y) = \left| \frac{dx}{dy} \right| f(x)$$

$$y = 2x + 1$$

$$2x = y - 1$$

$$x = \frac{y-1}{2}$$

$$\frac{dx}{dy} = \frac{1}{2}$$

$$f(y) = \frac{1}{2} e^{-x}$$

$$= \frac{1}{2} e^{-\left(\frac{y-1}{2}\right)}$$

Limit:	$x = 0$	$x = \infty$
	$y = 2x + 1$	$y = \infty$
	$y = 1$	

$$f(y) = \frac{1}{2} e^{-\left(\frac{y-1}{2}\right)}, \quad 1 < y < \infty$$

3) If x is uniformly distributed random variable in $(-\pi/2, \pi/2)$. Find pdf of $y = \tan x$

$$f(x) = \frac{1}{b-a}, \quad a < x < b$$

$$= \frac{1}{\pi/2 - (-\pi/2)}, \quad -\pi/2 < x < \pi/2$$

$$= \frac{1}{\pi}, \quad -\pi/2 < x < \pi/2$$



DEPARTMENT OF MATHEMATICS

$y = \tan x$
 $x = \tan^{-1} cy$
 $\frac{dx}{dy} = \frac{1}{1+y^2}$
 $f(y) = \left| \frac{dx}{dy} \right| f(x)$
 $f(y) = \frac{1}{\pi} \cdot \frac{1}{1+y^2}$
 Limits: $x = -\pi/2$ $x = \pi/2$
 $y = \tan(-\pi/2)$ $y = \tan(\pi/2)$
 $y = -\infty$ $y = \infty$
 $y = -\infty$
 $f(y) = \frac{1}{\pi} \cdot \frac{1}{1+y^2}$, $-\infty < y < \infty$

A) If pdf of x is $f(x) = 2x$, $0 < x < 1$. Find pdf of $y = 3x+1$.
 $y = 3x+1$
 $3x = y-1$
 $x = \frac{y-1}{3}$
 $\frac{dx}{dy} = \frac{1}{3}$
 $f(y) = \frac{1}{3} \cdot 2x$
 $= \frac{2}{3} \left(\frac{y-1}{3} \right)$, $1 < y < 4$
 $x=0$ $x=1$
 $y=1$ $y=4$