$= \lambda[-(-\lambda-1)(1)]_{t=0}$

$= \lambda(\lambda+1)$

$= \lambda^2+\lambda$

$Var(x) = E(x^2) - (E(x))^2$

$= \lambda^2+\lambda-\lambda^2$

$= \lambda$

problems

Normal distribution:

14^{th} %.

If x follows normal distribution, If its pdf is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

Mean $= \mu$, variance $= \sigma^2$, S.D $= \sigma$.

$$z = \frac{x-\mu}{\sigma}$$

Derive the MGF for Normal distribution and hence deduce its mean & variance.

$$M_x(t) = E(e^{tx})$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x)\,dx$$

$$= \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$z = \frac{x-\mu}{\sigma} \qquad \left\{ z = \frac{x-\mu}{\sigma} \right.$$

$$z\sigma = x-\mu$$

$$x = \mu + z\sigma$$

$$dx = 0 + \sigma dz$$

$$dx = \sigma dz$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\mu + z\sigma)} \cdot e^{-\frac{1}{2}z^2} \sigma dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\mu} e^{tz\sigma} e^{-\frac{1}{2}z^2} dz$$

$$= e^{t\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2 + tz\sigma} dz$$

$$= e^{t\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z^2 - 2tz\sigma)} dz$$

$$= e^{t\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(\underbrace{z^2 - 2tz\sigma + (t\sigma)^2}_{a^2 - 2ab + b^2} - (t\sigma)^2)} dz$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[(z - t\sigma)^2 - (t\sigma)^2]} dz$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[z - t\sigma]^2} e^{\frac{1}{2}(t\sigma)^2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \cdot e^{t\mu} \cdot e^{\frac{1}{2}(t\sigma)^2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[z - t\sigma]^2} dz$$

$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} \cdot e^{\frac{t^2\sigma^2}{2}} \cdot \sqrt{2\pi}$$

$$= e^{t\mu + \frac{t^2\sigma^2}{2}}$$

Mean:

$$E(X) = \frac{d}{dt}\left[M_X(t)\right]_{t=0}$$

$$= \frac{d}{dt}\left[e^{t\mu + \frac{t^2\sigma^2}{2}}\right]_{t=0}$$

$$= \left[e^{t\mu + \frac{t^2\sigma^2}{2}}\left(\mu + \frac{2t\sigma^2}{2}\right)\right]_{t=0}$$

$$= e^0 (0+\mu)$$

$$= \mu.$$

$$E(X^2) = \frac{d^2}{dt^2}\left[M_X(t)\right]_{t=0}$$

$$= \frac{d}{dt}\left[e^{t\mu + \frac{t^2\sigma^2}{2}}\left(\mu + t\sigma^2\right)\right]_{t=0}$$

$$= \left[e^{t\mu + \frac{t^2\sigma^2}{2}}(\sigma^2) + (\mu + t\sigma^2)\left(e^{t\mu + \frac{t^2\sigma^2}{2}}\left(\mu + t\sigma^2\right)\right)\right]_{t=0}$$

$$= e^0(\sigma^2) + (\mu+0)(e^0 \cdot (\mu+0))$$

$$= \sigma^2 + \mu^2$$

$Var(x) = E(x^2) - [E(x)]^2$

$= \sigma^2 + \mu^2 - (\mu)^2$

$Var(x) = \sigma^2$

$S.D = \sigma$

1. In a test on 2000 electric bulbs, it was found that the life of a particular bulb was normally distri with an ave life of 2040 hrs & S.D of 60 hr. Estimate the no of bulbs likely to burn for (i) more than 2150 hrs (ii) less than 1950 hrs m than 1920 hrs les 2160

1) The weekly wages of 1000 workers are normally distributed with mean Rs. 70 & S.D Rs. 5. Estimate the no. of workers whose weekly wages will be

i) less than 69.

ii) more than 72

iii) between 69 & 72.

0.0336 × 200 = 67

0.0668 = 134

0.9646 × 2000 = 1909

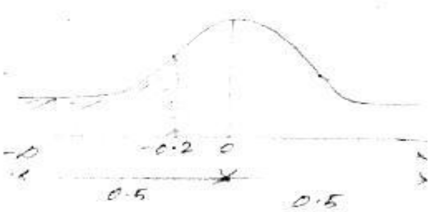
i) $P(\text{less than } 69) = P(X < 69)$

$= P\left(\frac{x-\mu}{\sigma} < \frac{69-\mu}{\sigma}\right)$

$= P\left(z < \frac{69-70}{5}\right)$

$= P(z < -1/5)$

$= P(z < -0.2)$



$= 0.5 - P(0 < z < 0.2)$

$= 0.5 - 0.0793$ (from table)

$= 0.4207$

Out of $1000 = 0.4207 \times 1000$

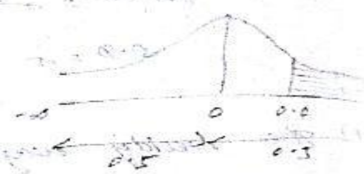
$= 420.7$

$\simeq 421$

ii) $P(\text{more than } 72) = P(X > 72)$

$$= P\left(\frac{X-\mu}{\sigma} > \frac{72-\mu}{\sigma}\right)$$

$$= P(z > 2/5)$$



$$= P(z > 0.4)$$

$$= 0.5 - P(0 < z < 0.4)$$

$$= 0.5 - 0.1554$$

$$= 0.3446.$$

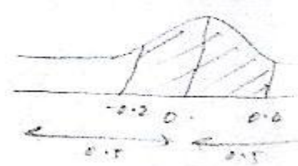
Out of $1000 = 0.3446 \times 1000$

$$= 345$$

iii) $P(69 < X < 72) = P\left(\frac{69-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{72-\mu}{\sigma}\right)$

$$= P(-1/5 < z < 2/5)$$

$$= P(-0.2 < z < 0.4)$$



$$= P(-0.2 < z < 0) + P(0 < z < 0.4)$$

$$= P(0 < z < 0.2) + P(0 < z < 0.4)$$

$$= 0.0793 + 0.1554$$

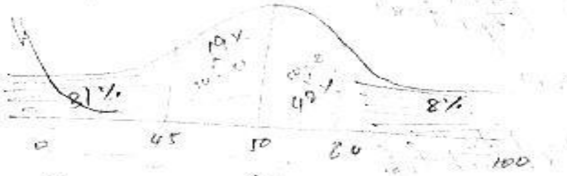
$$= 0.2347$$

Out of $1000 = 0.2347 \times 1000$

$$= 235$$

Ex type

2) In normal distribution, 31% of a items are under 45. 8% are above 64. Find mean and variance



$P(x<45) = 0.31$

$$P\left(\frac{x-\mu}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.31$$

$$P\left(z < \frac{45-\mu}{\sigma}\right) = 0.31$$

$$0.5 - P\left(0 < z < \frac{45-\mu}{\sigma}\right) = 0.31$$

$$P\left(0 < z < \frac{45-\mu}{\sigma}\right) = 0.19$$

$$\frac{45-\mu}{\sigma} = -0.5 \text{ (from table)} \to (1)$$

$$P(x>64) = 0.08$$

$$P\left(z > \frac{64-\mu}{\sigma}\right) = 0.08$$

$$P\left(0<z<\frac{64-\mu}{\sigma}\right) = 0.42$$

$$\frac{64-\mu}{\sigma} = +1.4 \to (2) \text{ (from table)}$$

$(1) \Rightarrow 45 - \mu = -0.5\sigma$

$(2) \Rightarrow 64 - \mu = +1.4\sigma$

$-19 = -1.9\sigma$

$\sigma = 19/1.9$

$$\boxed{\sigma = 10}$$

$-\mu = 1.4(10) - 64$

$-\mu = 14 - 64$

$\mu = 50$