

DEPARTMENT OF MATHEMATICS



10)

A continuous random variable 'X' is said to follow an exponential distribution with parameter x >0 if its probability density function (p.d.f) is given by, $f(\mathbf{x}) = \begin{cases} \alpha e^{-\alpha \mathbf{x}} , \ \alpha \ge 0 \\ 0 , \text{ otherwise} \end{cases}$ Moment generating function : $M_{x}(t) = E(e^{tx})$ $= \int e^{tx} f(x) dx$ = $\int_{0}^{\infty} e^{tx} de^{-\alpha x} dx$ [: x = 0] $= \alpha \int_{-e}^{\infty} \overline{e}^{(t+\alpha)x} dx$ $= \alpha \left[\frac{e^{Et+\alpha}}{e} \right]^{\infty}$ $= \frac{\alpha}{t \neq \alpha} \left[\frac{e^{\alpha}}{e^{\alpha}} - \frac{e^{\alpha}}{e^{\alpha}} \right]$ $= \frac{\alpha}{-1 \pm \frac{1}{2} \alpha} \left[0 - 1 \right]$ $= + \alpha$

16MA301- PROBABILITY AND QUEUEING THEORY

S.GOWRI/AP/MATHEMATICS

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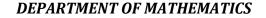
 $M_{\chi}(t) = \frac{\alpha}{\alpha - t}$

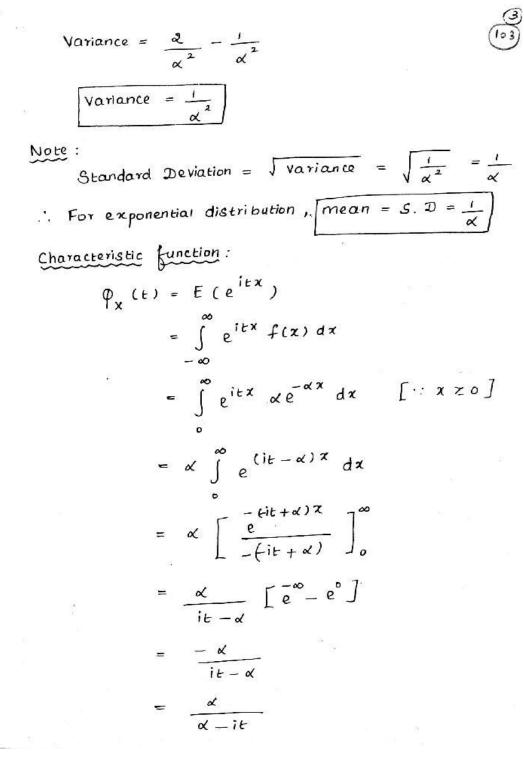
Dividing numerator and denominator by 'a', we get, $M_{x}(t) = \frac{1}{1-\frac{t}{\alpha}} \qquad M_{x}(t) = \alpha(\alpha-t)^{-1}$ $M_{x}(\alpha) = \alpha(-1)(\alpha-t)^{-2}(-1)^{-1}$ $= \frac{\alpha}{(\alpha-t)^{-1}} \qquad M_{x}'(\alpha) = \frac{\alpha}{\alpha^{-1}} = \frac{1}{\alpha}$ $= 1 + \frac{t}{\alpha} + \frac{t^{2}}{\alpha^{2}} + \dots + \frac{t}{\alpha^{\gamma}} + \dots$ $M_{\chi}(t) = \frac{\infty}{Y = 0} \left(\frac{t}{\alpha}\right)^{\gamma} \qquad M_{\chi}^{"}(t) = \omega \chi (\alpha - t)^{-3}$ $= 3\alpha (\alpha - t)^{-3}$ an and variance: $M_{\chi}^{"}(0) = \frac{3\alpha}{\kappa^{3}} = \frac{3}{\kappa^{1}}$ Mean and variance : = Coefficient of $\frac{t^{\gamma}}{\gamma I}$ in $M_{\chi}(t) = \frac{1}{\chi^2}$ $= \frac{\gamma!}{\alpha^{\gamma}}, \gamma = 1, 2, \cdots$ $Mean = \mu'_1 = \frac{1}{\alpha}$ $\mu_{2}^{\prime} = \frac{2!}{\sqrt{2}}$ Variance = $\mu_2' - (\mu_1')^2$

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$$J = 1, \text{ Variance > Mean}$$

$$x = 1, \text{ Variance = Mean}$$

$$x > 1, \text{ Variance = Mean}$$

$$x > 1, \text{ Variance < Mean}$$
Hence for an exponential distribution,
Variance > =, or < Mean for different values
of the parameter.
Exponential distribution lacks memory:

$$J = 1 \text{ K is exponentially distributed with parameter}$$

$$x, \text{ then for any two positive integens 's' and 't',}$$

$$P = [x > 3 + t | x > 3] = P [x > t]$$
Proof:

$$The p.d.b of x \text{ is },$$

$$f(x) = \begin{cases} xe^{-\alpha x}, x \ge 0\\ 0, \text{ otherwise} \end{cases}$$
Consider $P [x > t] = \int_{0}^{\infty} f(x) dx$

$$= \int_{0}^{\infty} de^{-\alpha x} dx$$

$$= \alpha \left[\frac{e^{-\alpha x}}{-\alpha} \right]_{t}^{\infty}$$

$$= - \left[e^{-\infty} - e^{-\alpha t} \right]$$





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$$P(x > t) = e^{-\alpha t} \longrightarrow 0$$
Consider,

$$P[x > \delta + t | x > \delta] = P[x > \delta + t | x > \delta]$$

$$= \frac{P(x > \delta + t)}{P(x > \delta)}$$

$$= \frac{e^{-\alpha (\delta + t)}}{e^{-\delta \alpha}} (using 0)$$

$$= e^{-\delta t} e^{-\alpha t} e^{-\delta t}$$

$$= e^{-\alpha t} \longrightarrow 0$$
From (1) and (2),

$$P[x > \delta + t | x > \delta] = P[x > t]$$
Thus exponential distribution lacks memory.

16MA301- PROBABILITY AND QUEUEING THEORY

1

 The mileage which car owners get with certain kind of radial type is a random variable having an exponential distribution with mean 4000 km. Find the
 probabilities that one of these tires will last

 (i) atleast 2000 km
 (ii) atleast 2000 km
 (iii) atleast 3000 km

[Memoryless Property of Exponential Distribution

Griven: $\alpha_{1} = \frac{1}{4000}$ Mean = $\frac{1}{4} = \frac{1}{4000}$ Then $f(x) = \alpha e^{-\alpha x}$ $f(x) = \frac{1}{4000} e^{-\chi/4000}$, x > 0

(i) Atleast 2000 km
P [x > 2000] =
$$\int_{-\infty}^{\infty} f(x) dx$$

2000
= $\int_{-\infty}^{\infty} \frac{1}{4000} e^{-x/4000} dx$
2000
 $= -\frac{x}{4000} - \frac{x}{4000} dx$

$$= \frac{1}{4000} \left[\frac{e}{\frac{-1}{4000}} \right]_{2000}$$
$$= - \left[e^{\infty} - e^{-\frac{1}{2}} \right]_{2000}$$

Problems :

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$$P[x > 2000] = e^{-0.5}$$

$$P[x > 2000] = 0.6665$$
(ii) Atmost 3000 km:

$$P(x \le 3000) = \int f(x) dx$$

$$= \int_{0}^{3000} e^{-x/4000} dx$$

$$= \int_{-1/4000}^{-1/4000} e^{-x/4000} dx$$

$$= - [-e^{0} + e^{-3/4}]$$

$$= 1 - e^{-0.45}$$

$$P(x \le 3000) = 0.5270$$
(2) For an exponential distribution with mean 120 days,
find the probability that such a watch will
(i) have to be set in less than 24 days and
(ii) not have to be reset in atleast 180 days
Solution:
Given : Mean = $\frac{1}{x} = 120$
 $\Rightarrow x = \frac{1}{120}$

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