CONTINUOUS DISTRIBUTIONS

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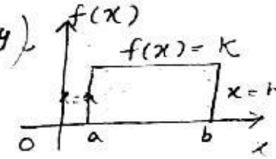
If X is a continuous random variable, then we have the following distributions.

1. Uniform (Rectangular) Distribution
2. Exponential Distribution
3. Gamma Distribution
4. Weibull Distribution.

UNIFORM (RECTANGULAR) DISTRIBUTIONS:

(Defn: A random variable 'X' is said to have a Continuous uniform distribution if its p.d.f is given by)



$$f(x) = \begin{cases} K, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

Where 'a' & 'b' are the two parameters of the uniform distribution. For a uniform distribution in (a, b)

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

Note:

1. The distribution function $F(x)$ is given by,

$$F(x) = \begin{cases} 0, & -\infty < x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b < x < \infty \end{cases}$$

2. The p.d.f of a uniform variate 'X' in $(-a, a)$ is given by,

$$f(x) = \begin{cases} \frac{1}{2a}, & -a < x < a \\ 0, & \text{Otherwise.} \end{cases}$$

Moment generating function:

$$M_x(t) = E(e^{tx})$$

$$= \int_a^b e^{tx} f(x)\, dx$$

$$= \int_a^b e^{tx} \cdot \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\left[\frac{e^{tx}}{t}\right]_a^b = \frac{e^{bt} - e^{at}}{t(b-a)}$$

$$\boxed{M_x(t) = \frac{e^{bt} - e^{at}}{(b-a)t}}$$

Mean and Variance:

$$\mu_r' = \int_a^b x^r f(x)\, dx$$

Put $r = 1$,

$$\mu_1' = \int_a^b x f(x)\, dx$$

$$= \int_a^b x \cdot \frac{1}{b-a}\, dx = \frac{1}{b-a}\int_a^b x\, dx$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{1}{2(b-a)}(b^2 - a^2)$$

$$= \frac{(b+a)\cancel{(b-a)}}{2\cancel{(b-a)}}$$

$$\boxed{\text{Mean} = \mu_1' = \frac{b+a}{2}}$$

Put $r = 2$,

$$\mu_2' = \int_a^b x^2 f(x)\, dx$$

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$$\mu_2' = \int_a^b x^2 \cdot \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} \int_a^b x^2\, dx = \frac{1}{b-a}\left[x^3/3\right]_a^b$$

$$= \frac{1}{3(b-a)}\left[b^3 - a^3\right] = \frac{\cancel{(b-a)}(b^2+ab+a^2)}{3\cancel{(b-a)}}$$

$$\boxed{\mu_2' = \frac{a^2+ab+b^2}{3}}$$

$$\text{Variance} = \mu_2' - \mu_1'^2$$

$$= \frac{a^2+ab+b^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{a^2+ab+b^2}{3} - \frac{a^2+2ab+b^2}{4}$$

$$= \frac{4a^2+4ab+4b^2-3a^2-6ab-3b^2}{12}$$

$$= \frac{a^2-2ab+b^2}{12}$$

$$\boxed{\text{Variance} = \frac{(b-a)^2}{12}}$$

Problems :

① Show that for a uniform distribution in (a, b)

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

Solution :

Since the total probability is 1, we have

$$\int^b f(x)\,dx = 1$$

CORRELATION

$$\int_a^b K\,dx = 1$$

$$K\,[x]_a^b = 1$$

$$K\,(b-a) = 1$$

$$K = \frac{1}{b-a}$$

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

7) (2) The number of Personal computer sold daily at a Compuworld is uniformly distributed with a minimum of 2000 pc and a maximum of 5000 PC. Find

(i) The probability that daily sales will fall between 2500 and 3000 pc.

(ii) What is the probability that the Compuworld will sell atleast 4000 PC's?

(iii) What is the probability that the compuworld will exactly sell 2500 PC's?

Solution:

Let X be the R.v denoting the number of computer sold daily at a Compuworld.

The pdf of a uniform distribution is,

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

Here $a = 2000$, $b = 5000$

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$$\therefore f(x) = \begin{cases} \frac{1}{5000-2000}, & 2000 < x < 5000 \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{1}{3000}, & 2000 < x < 5000 \\ 0, & \text{otherwise} \end{cases}$$

(i) $P(2500 < x < 3000) = \int_{2500}^{3000} f(x)\,dx$

$$= \int_{2500}^{3000} \frac{1}{3000}\,dx = \frac{1}{3000}\left[x\right]_{2500}^{3000}$$

$$= \frac{500}{3000} = \frac{1}{6} = 0.166$$

$$\boxed{P(2500 < X < 3000) = 0.166}$$

(ii) P(Selling atleast 4000 Pc's)

$$= P(x \geq 4000)$$

$$= \int_{4000}^{5000} f(x)\,dx$$

$$= \int_{4000}^{5000} \frac{1}{3000}\,dx = \frac{1}{3000}\left[x\right]_{4000}^{5000}$$

$$= \frac{5000-4000}{3000} = \frac{1000}{3000} = \frac{1}{3} = 0.33$$

$$\boxed{P(X \geq 4000) = 0.33}$$

(iii) P(Selling exactly 2500 pc's)

$$= P(X = 2500)$$

$$= 0 \qquad (\because P(x=c) = 0)$$