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#### **DEPARTMENT OF MATHEMATICS**

DISCRETE DISTRIBUTIONS :

The important discrete distributions of a random Variable 'x' are,

- 1. Binomial distribution
- 2. Poisson distribution
- 3. Geometric distribution
- 4. Negative Binomial distribution.

### BINOMIAL DISTRIBUTION :

The probability mass function of a random varial 'X' which follows the binomial distribution is,

P(x=x) = ncx px qn-x , x=0,1,2,...n & p + qr = 1(a+p)" = q" + n q q" - 1 p" + n g q" - 2 p2 + .... nc, px qn-x

which is a binomial series and hence the distribution is called a Binomial distribution.

NOTE :

 $P(o \ Success) = nc_{o} p^{o} q^{n-o} = q^{n}$ P(I Success) = nc, p'q'-1  $P(2 \text{ Success}) = nc_2 p^2 q^{m-2}$  and so on.

ASSUMPTIONS :

- (i) There are only two possible outcomes for each trial (Success or failure)
- (ii) The probability of a success is the same for each tria (iii) There are 'n' trials, where 'n' is a constant.
- (iv) The 'n' trials are independent.

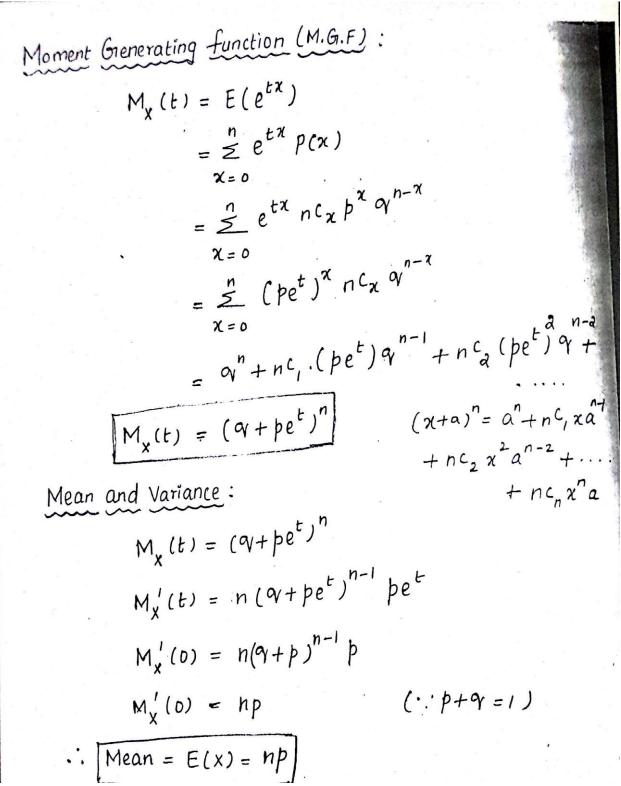
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$$M_{x}^{*}(t) = np \left[ (\alpha_{t} + pe^{t})^{n-t} e^{t} + e^{t} (n-t)(\alpha_{t} + pe^{t})^{n-t} + (n-t)(\alpha_{t} + p)^{n-t} p \right]$$

$$= np \left[ (\alpha_{t} + p)^{n-t} + (n-t)(\alpha_{t} + p)^{n-t} p \right]$$

$$= np + n^{2}p^{2} - np^{2}$$

$$M_{x}^{*}(0) = n^{2}p^{2} + np(t-p)$$

$$M_{x}^{*}(0) = E(x^{2}) = n^{2}p^{2} + np\alpha$$

$$\therefore \text{ Vaxiance } E(x^{2}) - \left[ E(x) \right]^{2}$$

$$= n^{2}p^{2} + np\alpha - (np)^{2}$$

$$= n^{2}p^{2} + np\alpha - n^{2}p^{2}$$

$$\boxed{\text{Variance } = np\alpha}$$

$$\text{Standard deviation } = \sqrt{\text{Variance}}$$

$$\boxed{\text{SD} = \sqrt{np^{2}}}$$



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PROBLEMS : D The mean and SD of a binomial distribution are 20 and 4. Determine the distribution. Solution : Mean = np = 20 -> 1  $SD = \sqrt{npq} = 4$  $hpq = 4^2 = 16 \longrightarrow 2$  $\begin{array}{c} \textcircled{2} \\ \hline \end{array} = \begin{array}{c} h \not p & v \\ \hline h \not p \end{array} = \begin{array}{c} 16 \\ \hline 20 \end{array} \implies \begin{array}{c} 0 v = \frac{4}{5} \\ \hline 5 \end{array} \end{array}$  $p = 1 - q' = 1 - \frac{1}{5} = \frac{1}{5}$  $p = \frac{1}{5}$ subs p in (1),  $\frac{h \cdot l}{5} = 20$ n = 100 . The binomial distribution is ,  $P(x = x) = p(x) = n(x p^{x} q^{n-x})$  $= 100^{\circ} \chi \left(\frac{1}{5}\right)^{2} \left(\frac{4}{5}\right)^{100-x}, \chi = 0, 1, \dots 100$ If X is a binomial random variable with expected (2)value & and variance 4/3, find P(x=5). Solution : E(x) = 2 $np = 2 \longrightarrow 0$ Variance =  $\frac{1}{3}$ hpq= 4/3 -> 2

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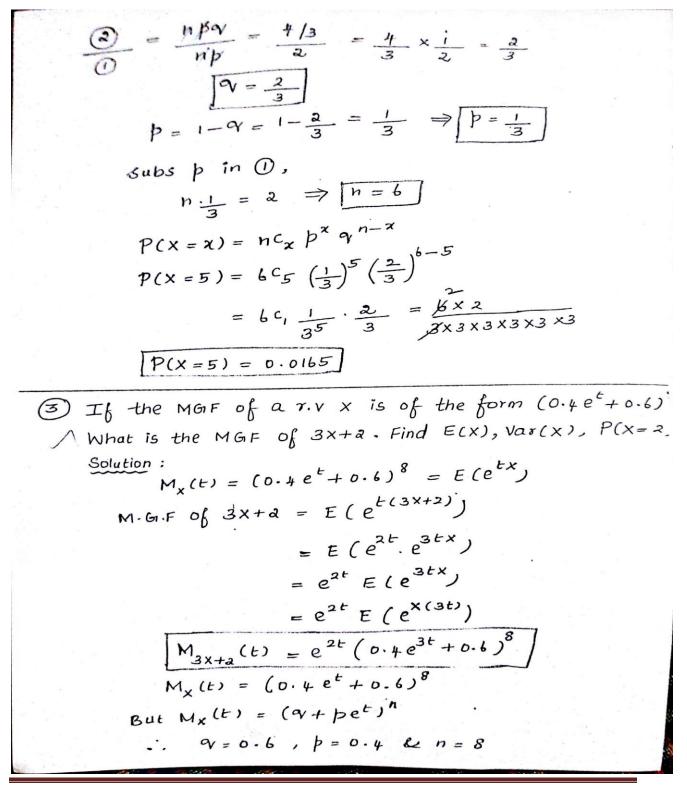
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$$E(x) = np = 8 \times 0.4 = 3.2$$

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Variance =  $E(x/r) + npq = 8 \times 0.4 \times 0.$ 

$$Variance = 1.9a$$

$$p(x = x) = nc_x p^x q^{n-x}$$

$$P(x = a) = 8c_a (0.4)^a (0.6)^{8-a}$$

$$= \frac{8 \times 7}{a} \times 0.16 \times 0.0467$$

$$P(x = a) = 0.209a$$

If 10 % of the screws produced by an automatic machine are defective, find the probability that Out of 20 screws selected at random, there are (i) exactly 2 defective (ii) atmost 3 defective (iii) atleast 2 defectives and (iv) between 1 and 3 defectives (inclusive).

Let X be the R.V denoting the number of defective screws.

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$$P(getting exactly a defectives) = 0.2852$$
(i)  $P(getting atmost 3 defective) = P(x \le 3)$ 

$$= P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$$

$$= 20 C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20-0} + 20 C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{20-1} + 20 C_2 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^{20-3}$$

$$= \left(\frac{9}{10}\right)^{20} + \frac{30}{10} \left(\frac{9}{10}\right)^{19} + \frac{20 \times 19}{2} \times \frac{1}{100} \left(\frac{9}{10}\right)^{16} + \frac{20 \times 19 \times 18}{2 \times 3} \times \frac{1}{1000} \left(\frac{9}{10}\right)^{16} + \frac{20 \times 19 \times 18}{2 \times 3} \times \frac{1}{1000} \left(\frac{9}{10}\right)^{16} + \frac{20 \times 19 \times 18}{2 \times 3} \times \frac{1}{1000} \left(\frac{9}{10}\right)^{16}$$

$$= 0.1216 + 0.2702 + 0.2852 + 0.1901$$

$$= 0.8671$$
(iii)  $P(getting at least a defective =  $P(x \ge 2)$ 

$$= 1 - [P(x < 2)]$$

$$= 1 - [P(x < 2)]$$

$$= 1 - [P(x < 2) + P(x = 1) + P(x = 2)]$$

$$= 0.323$$
(iv)  $P(getting number of defectives between 1 and 3)$ 

$$= P(1 \le x \le 3)$$

$$= 0.2702 + 0.2852 + 0.1901$$$ 

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