UNIT - III ①

TWO DIMENSIONAL RANDOM VARIABLES

Definition: Let S be the sample space of a random experiment. Let X and Y be two random variables defined on S. Then the pair (X, Y) is called a two-dimensional random variable or a bivariate random variable.

Types of Two-dimensional random Variables:

1. Discrete random variable
2. Continuous random variable.

1. Discrete random variable:

If the possible values of (X, Y) are finite, then (X, Y) is called a two-dimensional discrete random variable and it can be represented by (x_i, y_j) where $i = 1, 2, \ldots n$ and $j = 1, 2, \ldots m$

Joint probability Distribution of (X, Y):

Let (X, Y) be a two-dimensional discrete random variable. Let $P(X = x_i / Y = y_j) = p_{ij}$. p_{ij} is called the joint probability distribution of (X, Y) if the following conditions are satisfied:

(i) $p_{ij} \geq 0$ for all i and j

(ii) $\sum_j \sum_i p_{ij} = 1$

PROBLEMS :

① If the joint pdf of (X, Y) is given by
$P(x,y) = k(2x+3y)$, $x = 0,1,2$; $y = 1,2,3$.
Find all the marginal probability distribution. Also find the Probability distribution of $(X+Y)$ and $P(X+Y > 3)$

Solution :

Given : $p(x,y) = K(2x+3y)$

$p(0,1) = K(0+3) = 3K$

$p(0,2) = K(0+6) = 6K$

$p(0,3) = K(0+9) = 9K$

$p(1,1) = K(2+3) = 5K$

$p(1,2) = K(2+6) = 8K$

$p(1,3) = K(2+9) = 11K$

$p(2,1) = K(4+3) = 7K$

$p(2,2) = K(4+6) = 10K$

$p(2,3) = K(4+9) = 13K$

The marginal distributions are given in the table :

Y \ X	0	1	2	$\sum_x p(x,y)$
1	3K	5K	7K	15K
2	6K	8K	10K	24K
3	9K	11K	13K	33K
$\sum_y p(x,y)$	18K	24K	30K	72K

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$$\sum_y \sum_x p(x,y) = 1$$

$$72K = 1$$

$$\boxed{K = \frac{1}{72}}$$

Marginal distribution of X and Y :

X :	0	1	2
p(x) :	$\frac{18}{72}$	$\frac{24}{72}$	$\frac{30}{72}$

Y :	1	2	3
p(y) :	$\frac{15}{72}$	$\frac{24}{72}$	$\frac{33}{72}$

Probability distribution of X+Y :

$$P(X+Y=1) = p(0,1) = 3K = \frac{3}{72}$$

$$P(X+Y=2) = p(1,1) + p(0,2) = 5K + 6K = 11K = \frac{11}{72}$$

$$P(X+Y=3) = p(2,1) + p(1,2) + p(0,3)$$

$$= 7K + 8K + 9K$$

$$= 24K$$

$$= \frac{24}{72}$$

$$P(X+Y=4) = p(1,3) + p(2,2) = 11K + 10K = 21K = \frac{21}{72}$$

$$P(X+Y=5) = p(2,3) = 13K = \frac{13}{72}$$

$P(X+Y>3)$:

$$P(X+Y>3) = P(X+Y=4) + P(X+Y=5) = \frac{21}{72} + \frac{13}{72} = \frac{34}{72}$$