Matrix inversion by Gauss Jordan method.

Let A be a non singular square matrix then X is said to be the inverse of A if $AX = I$.

1) Find the inverse of a matrix by gauss Jordan method

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

Sol:

Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

Let $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The augmented matrix is given by

$$[A : I] = \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right)$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 2R_1$

$$[A : I] \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & -3 & -2 & 0 & 1 \end{array}\right)$$

$R_3 \to R_3 - R_2$

$$[A : I] \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -4 & -1 & -1 & 1 \end{array}\right)$$

$R_1 \to R_1 - R_2$

$$(A:I) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & -2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -4 & -1 & -1 & 1 \end{array}\right)$$

$R_1 \to 4R_1 + R_3$

$R_2 \to 4R_2 + R_3$

$$(A:I) \sim \left(\begin{array}{ccc|ccc} 4 & 0 & 0 & 7 & -5 & 1 \\ 0 & 4 & 0 & -5 & 3 & 1 \\ 0 & 0 & -4 & -1 & -1 & 1 \end{array}\right)$$

$R_1 \to \frac{R_1}{4}$

$R_2 \to \frac{R_2}{4}$

$R_3 \to \frac{R_3}{-4}$

$$(A:I) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/4 & -5/4 & 1/4 \\ 0 & 1 & 0 & -5/4 & 3/4 & 1/4 \\ 0 & 0 & 1 & 1/4 & 1/4 & -1/4 \end{array}\right)$$

$$A^{-1} = \frac{1}{4}\begin{pmatrix} 7 & -5 & 1 \\ -5 & 3 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

1) Find the inverse of $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ Ans $A^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

2) Find the inverse of $\begin{pmatrix} 3 & -1 & -1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ Ans $A^{-1} = \begin{pmatrix} 2 & 4 & 11 \\ 5 & 11 & 30 \\ 0 & 1 & 3 \end{pmatrix}$