④

② The two dimensional random variable (X, Y) has the joint probability mass function $f(x,y) = \dfrac{x+2y}{27}$, $x = 0, 1, 2$; $y = 0, 1, 2$. Find the conditional distribution of Y for $X = x$. Also find the conditional distribution of Y given $X = 1$

Solution:

The conditional distribution of Y for $x = x$ is.

$$f(y|x) = \frac{f(x,y)}{f(x)} \longrightarrow ①$$

where $f(x,y)$ is the joint probability function of X and Y.

Given : $f(x,y) = \dfrac{x+2y}{27}$

$f(0,0) = \dfrac{0+0}{27} = 0$

$f(0,1) = \dfrac{0+2}{27} = \dfrac{2}{27}$

$f(0,2) = \dfrac{0+4}{27} = \dfrac{4}{27}$

$f(1,0) = \dfrac{1+0}{27} = \dfrac{1}{27}$

$f(1,1) = \dfrac{1+2}{27} = \dfrac{3}{27}$

$f(1,2) = \dfrac{1+4}{27} = \dfrac{5}{27}$

$f(2,0) = \dfrac{2+0}{27} = \dfrac{2}{27}$

$f(2,1) = \dfrac{2+2}{27} = \dfrac{4}{27}$

$f(2,2) = \dfrac{2+4}{27} = \dfrac{6}{27}$

Y \ X	0	1	2	$P(Y=y)$
0	0	1/27	2/27	3/27
1	2/27	3/27	4/27	9/27
2	4/27	5/27	6/27	15/27
$P(X=x)$	6/27	9/27	12/27	1

Conditional distribution of Y for $x = x$:

When $x = 0$,

$$P(Y=0/X=0) = \frac{P(X=0, Y=0)}{P(X=0)} = \frac{0}{6/27} = 0$$

$$P(Y=1/X=0) = \frac{P(X=0, Y=1)}{P(X=0)} = \frac{2/27}{6/27} = \frac{1}{3}$$

$$P(Y=2/X=0) = \frac{P(X=0, Y=2)}{P(X=0)} = \frac{4/27}{6/27} = \frac{2}{3}$$

When $x = 1$,

$$P(Y=0/X=1) = \frac{P(X=1, Y=0)}{P(X=1)} = \frac{1/27}{9/27} = \frac{1}{9}$$

$$P(Y=1/X=1) = \frac{P(X=1, Y=1)}{P(X=1)} = \frac{3/27}{9/27} = \frac{1}{3}$$

$$P(Y=2/X=1) = \frac{P(X=1, Y=2)}{P(X=1)} = \frac{5/27}{9/27} = \frac{5}{9}$$

When $x = 2$,

$$P(Y=0/X=2) = \frac{P(X=2, Y=0)}{P(X=2)} = \frac{2/27}{12/27} = \frac{1}{6}$$

$$P(Y=1/X=2) = \frac{P(X=2, Y=1)}{P(X=2)} = \frac{4/27}{12/27} = \frac{1}{3}$$

$$P(Y=2/X=2) = \frac{P(X=2, Y=2)}{P(X=2)} = \frac{6/27}{12/27} = \frac{1}{2}$$