Function of random variable: [Transformation of one dimensional random variable]

$$f(y) = \left|\frac{dx}{dy}\right| f(x)$$

(or)

$$f_Y(y) = \left|\frac{dx}{dy}\right| f_X(x)$$

1) Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x/2 & , 1 < x < 5 \\ 0 & , \text{otherwise} \end{cases}$$

. Find pdf of $y = 2x - 3$

$$f(y) = \left|\frac{dx}{dy}\right| f(x)$$

$y = 2x - 3$

$2x = y + 3$

$\boxed{x = \frac{y+3}{2}}$

$\frac{dx}{dy} = \frac{1}{2}(1)$

$f(y) = \frac{1}{2} \cdot \frac{x}{2}$

$= \frac{1}{4}\left(\frac{y+3}{2}\right)$

$= \frac{1}{8}(y+3)$

$f(y) = \frac{y+3}{8} \quad , -1 < y < 7$

Limit: $x = 1$ $\quad$ $x = 5$

$y = 2 - 3$ $\quad$ $y = 10 - 3$

$y = -1$ $\quad$ $y = 7$

2) If the pdf of x is $f(x) = e^{-x}$, $x > 0$. Find the pdf of $y = 2x+1$.

$0 < x < \infty$

$$f(y) = \left|\frac{dx}{dy}\right| f(x)$$

$$y = 2x+1$$

$$2x = y-1$$

$$\boxed{x = \frac{y-1}{2}}$$

$$\frac{dx}{dy} = \frac{1}{2}$$

$$f(y) = \frac{1}{2} e^{-x}$$

$$= \frac{1}{2} e^{-\left(\frac{y-1}{2}\right)}$$

Limit: $x = 0$ $\qquad$ $x = \infty$

$y = 2x+1$ $\qquad$ $y = \infty$

$y = 1$

$$f(y) = \frac{1}{2} e^{-\left(\frac{y-1}{2}\right)}, \quad 1 < y < \infty$$

3) If x is uniformly distributed random variable in $(-\pi/2, \pi/2)$. Find pdf of $y = \tan x$

$$f(x) = \frac{1}{b-a}, \quad a < x < b$$

$$= \frac{1}{\pi/2 + \pi/2}, \quad -\pi/2 < x < \pi/2$$

$$= \frac{1}{\pi}, \quad -\pi/2 < x < \pi/2$$

$y = \tan x$.

$x = \tan^{-1}(y)$

$\frac{dx}{dy} = \frac{1}{1+y^2}$

$f(y) = \left|\frac{dx}{dy}\right| f(x)$

$f(y) = \frac{1}{\pi} \cdot \frac{1}{1+y^2}$

limit: $x = -\pi/2$ $\qquad x = \pi/2$

$y = \tan(-\pi/2)$ $\qquad y = \tan(\pi/2)$

$y = -\tan \pi/2$ $\qquad y = \infty$

$y = -\infty$

$f(y) = \frac{1}{\pi} \cdot \frac{1}{1+y^2}, \quad -\infty < y < \infty$

4) If pdf of x is $f(x) = 2x,\ 0<x<1$. Find pdf of $y = 3x+1$.

$y = 3x+1$

$3x = y-1$

$x = \frac{y-1}{3}$

$\frac{dx}{dy} = \frac{1}{3}$

$f(y) = \frac{1}{3} \cdot 2x$.

$= \frac{2}{3}\left(\frac{y-1}{3}\right), \quad 1<y<4$

$x=0 \qquad x=1$

$y=1 \qquad y=4$.