



DEPARTMENT OF MATHEMATICS

$$= \lambda [-(\lambda - 1)(1)]_{\lambda=0}^{\infty}$$

$$= \lambda(\lambda + 1)$$

$$= \lambda^2 + \lambda$$

$$\text{Var}(x) = E(x^2) - (E(x))^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$= \lambda$$

problem: Normal distribution:
 If x follows normal distribution, its pdf is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

Mean = μ , variance = σ^2 , S.D = σ .

$$z = \frac{x - \mu}{\sigma}$$

derive the MGF for Normal distribution and hence deduce its mean & variance.

$$M_x(t) = E(e^{tx})$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x) dx$$

$$= \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$



$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$
$$z = \frac{x-\mu}{\sigma} \quad \left\{ \begin{array}{l} z = \frac{x-\mu}{\sigma} \\ z\sigma = x-\mu \\ x = \mu + z\sigma \\ dx = 0 + \sigma dz \\ dx = \sigma dz \end{array} \right.$$
$$z\sigma = x - \mu$$
$$x = \mu + z\sigma$$
$$dx = 0 + \sigma dz$$
$$dx = \sigma dz$$
$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\mu+z\sigma)} \cdot e^{-\frac{1}{2}z^2} \sigma dz$$
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\mu} e^{tz\sigma} \cdot e^{-\frac{1}{2}z^2} dz$$
$$= e^{t\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2 + tz\sigma} dz$$
$$= e^{t\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z^2 - 2tz\sigma)} dz$$
$$= e^{t\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z^2 - 2tz\sigma + (t\sigma)^2 - (t\sigma)^2)} dz$$
$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[(z-t\sigma)^2 - (t\sigma)^2]} dz$$
$$= \frac{1}{\sqrt{2\pi}} e^{t\mu} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-t\sigma)^2} e^{\frac{1}{2}(t\sigma)^2} dz$$
$$= \frac{1}{\sqrt{2\pi}} \cdot e^{t\mu} \cdot e^{\frac{1}{2}(t\sigma)^2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-t\sigma)^2} dz$$



DEPARTMENT OF MATHEMATICS

$$= \frac{1}{\sqrt{2\pi}} e^{-t\mu} \cdot e^{-\frac{t^2\sigma^2}{2}} \cdot \sqrt{2\pi}$$

$$= e^{-t\mu + \frac{t^2\sigma^2}{2}}$$

Mean:

$$E(X) = \frac{d}{dt} [M_X(t)]_{t=0}$$

$$= \frac{d}{dt} \left[e^{-t\mu + \frac{t^2\sigma^2}{2}} \right]_{t=0}$$

$$= \left[e^{-t\mu + \frac{t^2\sigma^2}{2}} \left(-\mu + \frac{2t\sigma^2}{2} \right) \right]_{t=0}$$

$$= e^0 (-\mu)$$

$$= -\mu$$

$$E(X^2) = \frac{d^2}{dt^2} [M_X(t)]_{t=0}$$

$$= \frac{d}{dt} \left[e^{-t\mu + \frac{t^2\sigma^2}{2}} \left(\mu + \frac{2t\sigma^2}{2} \right) \right]_{t=0}$$

$$= \left[e^{-t\mu + \frac{t^2\sigma^2}{2}} \left(-\mu + \frac{2t\sigma^2}{2} \right) + (\mu + t\sigma^2) \left(e^{-t\mu + \frac{t^2\sigma^2}{2}} \left(-\mu + \frac{2t\sigma^2}{2} \right) \right) \right]_{t=0}$$

$$= e^0 (\sigma^2) + (\mu + 0) (e^0 \cdot (-\mu + 0))$$

$$= \sigma^2 + \mu^2$$



DEPARTMENT OF MATHEMATICS

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= \sigma^2 + \mu^2 - (\mu)^2 \end{aligned}$$

$$\text{Var}(X) = \sigma^2$$

$$\text{S.D} = \sigma$$

1. In a test on 2000 electric bulbs, it was found that the life of a particular bulb was normally distributed with an average of 2040 hrs & S.D of 60 hr. Estimate the no. of bulbs likely to burn for (i) more than 2150 hrs

(ii) less than 1950 hrs
(iii) more than 1920 hrs

1) The weekly wages of 1000 workers are normally distributed with mean Rs. 70 & S.D Rs. 5. Estimate the no. of workers whose weekly wages will be

i) less than 69.

$$0.0338 \times 200 = 67$$

ii) more than 72

$$0.0643 = 134$$

iii) between 69 & 72.

$$0.9656 \times 2000 = 1931$$

i) $P(\text{less than } 69) = P(X < 69)$

$$= P\left(\frac{X - \mu}{\sigma} < \frac{69 - \mu}{\sigma}\right)$$

$$= P\left(Z < \frac{69 - 70}{5}\right)$$

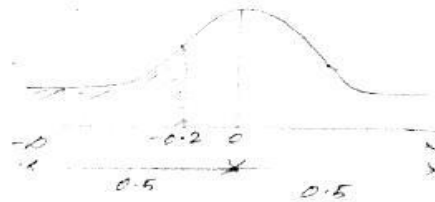
$$= P(Z < -0.2)$$

$$= P(Z < -0.2)$$

$$= 0.5 - P(0 < Z < 0.2)$$

$$= 0.5 - 0.0793 \text{ (from table)}$$

$$= 0.4207$$



$$\text{Out of } 1000 = 0.4207 \times 1000$$

$$= 420.7 \approx 421$$

$$\approx 421$$



DEPARTMENT OF MATHEMATICS

ii) $P(\text{more than } 72) = P(X > 72)$

$= P\left(\frac{X-M}{\sigma} > \frac{72-M}{\sigma}\right)$

$= P(Z > \frac{2}{5})$

$= P(Z > 0.4)$

$= 0.5 - P(0 < Z < 0.4)$

$= 0.5 - 0.1554$

$= 0.3446$

Out of 1000 = 0.3446×1000

$= 345$

iii) $P(69 < X < 72) = P\left(\frac{69-M}{\sigma} < \frac{X-M}{\sigma} < \frac{72-M}{\sigma}\right)$

$= P\left(\frac{1}{5} < Z < \frac{2}{5}\right)$

$= P(0.2 < Z < 0.4)$


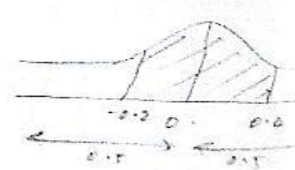
$= P(0 < Z < 0.2) + P(0 < Z < 0.4)$

$= 0.0793 + 0.1554$

$= 0.2347$

Out of 1000 = 0.2347×1000

$= 235$

2) In normal distribution, 31% of a Home are under 45, 8% are above 64. Find mean and variance.



$$P(X < 45) = 0.31$$

$$P\left(\frac{X-M}{\sigma} < \frac{45-M}{\sigma}\right) = 0.31$$

$$P(Z < \frac{45-M}{\sigma}) = 0.31$$

$$0.5 - P(0 < Z < \frac{45-M}{\sigma}) = 0.31$$

$$P(0 < Z < \frac{45-M}{\sigma}) = 0.19$$

$$\frac{45-M}{\sigma} = -0.5 \text{ (from table)} \rightarrow \textcircled{1}$$

$$P(X > 64) = 0.08$$

$$P(Z > \frac{64-M}{\sigma}) = 0.08$$

$$P(0 < Z < \frac{64-M}{\sigma}) = 0.42$$

$$\frac{64-M}{\sigma} = 1.4 \text{ (from table)} \rightarrow \textcircled{2}$$

$$\textcircled{1} \Rightarrow 45 - M = -0.5\sigma$$

$$\textcircled{2} \Rightarrow \frac{64 - M}{\sigma} = 1.4$$

$$-19 = -1.9\sigma$$

$$\sigma = \frac{19}{1.9}$$

$$\boxed{\sigma = 10}$$

$$-M = 1.4(10) - 64$$

$$-M = 14 - 64$$

$$M = 50$$