



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

NORMAL DISTRIBUTION



Normal distribution:

If x is a continuous random variable which follows normal distribution,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Note:

$$M_x(t) = e^{t\mu + \frac{t^2\sigma^2}{2}}$$

$$E(x) = \mu$$

$$\text{Var}(x) = \sigma^2$$

$$\text{SD} = \sqrt{\text{var}} = \sqrt{\sigma^2} = \sigma$$

Properties/characteristics of Normal Distribution

- * The curve is bell shaped and symmetric about the origin.
- * The curve has a single peak i.e., Unimodal.
- * The mean of the normal distribution lies at the centre of the curve.
- * In Normal distribution, mean, median & mode are coincide.

Note:

$$* z = \frac{x-\mu}{\sigma}$$

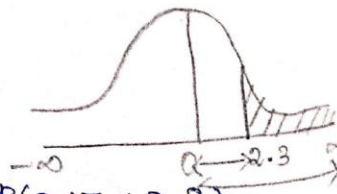
$$* P(-\infty < z < 0) = 0.5$$

$$* P(0 < z < \infty) = 0.5$$

Example:

J. $P(z > 2.3)$

$$\begin{aligned}
 P(z > 2.3) &= P(0 < z < \infty) - P(0 < z < 2.3) \\
 &= 0.5 - P(0 < z < 2.3)
 \end{aligned}$$





SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

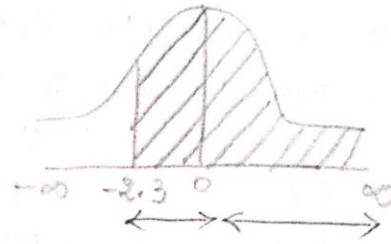
DEPARTMENT OF MATHEMATICS

NORMAL DISTRIBUTION



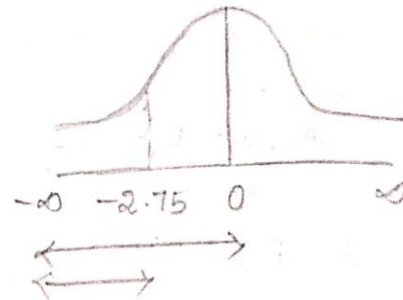
2]. $P(Z > -2.3)$

$$\begin{aligned} P(Z > -2.3) &= P(-2.3 < Z < 0) \\ &\quad + P(0 < Z < \infty) \\ &= P(0 < Z < 2.3) + 0.5 \end{aligned}$$



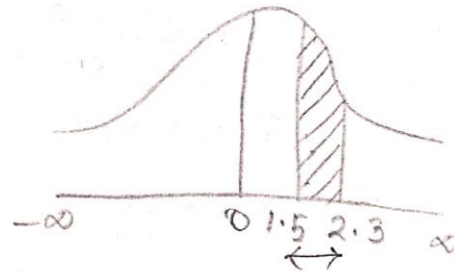
3]. $P(Z < -2.75)$

$$\begin{aligned} P(Z < -2.75) &= P(-\infty < Z < 0) \\ &\quad - P(-2.75 < Z < 0) \\ &= P(0 < Z < \infty) + P(0 < Z < 2.75) \\ &= 0.5 + P(0 < Z < 2.75) \end{aligned}$$



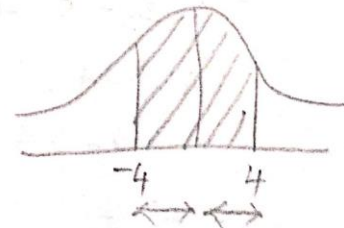
4]. $P(1.5 < Z < 2.3)$

$$\begin{aligned} P(1.5 < Z < 2.3) \\ &= P(0 < Z < 2.3) - P(0 < Z < 1.5) \end{aligned}$$



5]. $P(-4 < Z < 4)$

$$\begin{aligned} P(-4 < Z < 4) \\ &= P(-4 < Z < 0) + P(0 < Z < 4) \\ &= P(0 < Z < 4) + P(0 < Z < 4) \\ &= 2 P(0 < Z < 4) \end{aligned}$$





SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

NORMAL DISTRIBUTION



1. In a test on 2000 electric bulbs, it was found that the life of the particular make, was normally distributed with an ^{mean} average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for i). more than 2150 hours, ii). less than 1950 hours and iii). more than 1920 hours but less than 2160 hours.

Soln.:

$$\text{Mean: } \mu = 2040 \text{ hours}$$

$$\text{S.D.: } \sigma = 60 \text{ hours}$$

$$\text{i). } P(X \geq 2150)$$

$$\begin{aligned} \text{Let } Z &= \frac{X - \mu}{\sigma} \\ &= \frac{2150 - 2040}{60} \end{aligned}$$

$$Z = 1.833$$

$$\therefore P(X \geq 2150) = P(Z \geq 1.833)$$

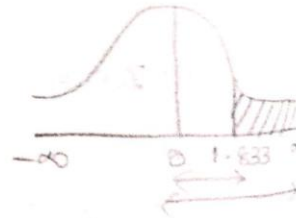
$$= P(0 < Z < \infty) - P(0 < Z < 1.833)$$

$$= 0.5 - 0.4664$$

$$= 0.034$$

$$= 0.34 \times 2000$$

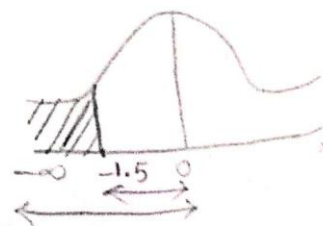
$$= 67.2$$



$$\text{ii). } P(X \leq 1950)$$

$$\begin{aligned} \text{Let } Z &= \frac{X - \mu}{\sigma} \\ &= \frac{1950 - 2040}{60} \end{aligned}$$

$$Z = -1.5$$





SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

NORMAL DISTRIBUTION



$$\begin{aligned}P(x \leq 1950) &= P(z \leq -1.5) \\&= P(-\infty < z < 0) - P(-1.5 < z < 0) \\&= P(0 < z < \infty) - P(0 < z < 1.5) \\&= 0.5 - 0.4332 \\&= 0.067 \\&= 0.067 \times 2000 \\&= 134\end{aligned}$$

iii) More than 1920 hrs and less than 2160 hrs.

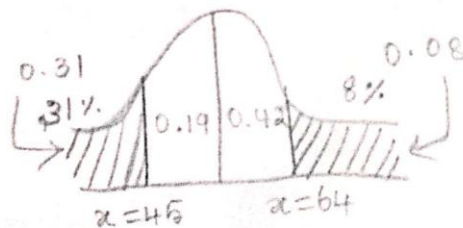
$$P(1920 \leq x \leq 2160)$$

$$\text{Let } x = 1920 \Rightarrow z = \frac{1920 - 2040}{60} = -2$$

$$x = 2160 \Rightarrow z = \frac{2160 - 2040}{60} = 2$$

$$\begin{aligned}P(1920 \leq x \leq 2160) &= P(-2 \leq z \leq 2) \\&= 2P(0 \leq z \leq 2) \\&= 2[0.4772] \\&= 0.954 \\&= 0.954 \times 2000 \\&= 1908\end{aligned}$$

5]. In a Normal distribution, 31% of the items are under 45 and 8% are over 64. Find mean & S.D.





SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

NORMAL DISTRIBUTION



$$\text{Let } z = \frac{x - \mu}{\sigma}$$

$$\text{When } x = 45$$

$$\text{0.19 table value } \frac{-0.5}{\sigma} = \frac{45 - \mu}{\sigma}$$

$$-0.5\sigma = 45 - \mu$$

$$\mu - 0.5\sigma = 45$$

↳ (1)

Solving (1) and (2),

$$-1.9\sigma = -19$$

$$\sigma = \frac{-19}{-1.9}$$

$$\sigma = 10$$

Subs. $\sigma = 10$ in (1)

$$\mu = 45 + 0.5\sigma$$

$$= 45 + 0.5(10)$$

$$= 45 + 5$$

$$\mu = 50$$

$$x = 64$$

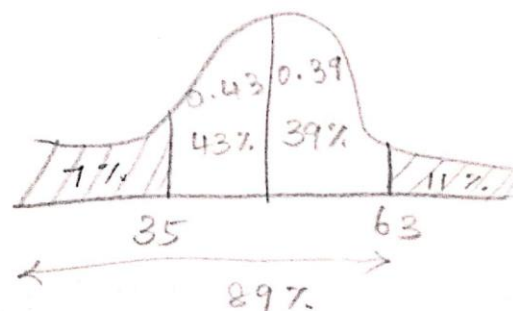
$$\text{0.42 table value } \frac{1.41}{\sigma} = \frac{64 - \mu}{\sigma}$$

$$1.41\sigma = 64 - \mu$$

$$\mu + 1.41\sigma = 64$$

↳ (2)

3]. In a distribution exactly normal 7% of one items are under 35 and 89% are under 63. What are the mean & SD of distribution.





SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

NORMAL DISTRIBUTION



when $x = 35$

$$z = \frac{x - \mu}{\sigma}$$

$$\textcircled{-1.48} = \frac{35 - \mu}{\sigma}$$

0.43 table value

$$\mu - 1.48\sigma = 35$$

↳ (1)

$x = 63$

$$z = \frac{x - \mu}{\sigma}$$

$$\textcircled{1.23} = \frac{63 - \mu}{\sigma}$$

0.39 table value

$$\mu + 1.23\sigma = 63$$

↳ (2)

Solving (1) & (2), we get

$$\mu = 50.29$$

$$\sigma = 10.33$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore – 641 035

DEPARTMENT OF MATHEMATICS

NORMAL DISTRIBUTION



AREA UNDER NORMAL CURVE TABLE OF AREAS

$\downarrow Z \rightarrow$	0	1	2	3	4	5	6	7	8	9
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0539	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0480	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4603	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4879	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4995	.4900	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4959	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4952
2.7	.4965	.4966	.4967	.4958	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4983	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000